

MOBILE JAMMERS FOR SECRECY RATE MAXIMIZATION IN COOPERATIVE NETWORKS

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ABSTRACT

We consider a source (Alice) trying to communicate with a destination (Bob), in a way that an unauthorized node (Eve) cannot infer, based on her observations, the information that is being transmitted. The communication is assisted by multiple multi-antenna cooperating nodes (helpers) who have the ability to move. While Alice transmits, the helpers transmit noise that is designed to affect the entire space except Bob. We consider the problem of selecting the helper weights and positions that maximize the system secrecy rate. It turns out that this optimization problem can be efficiently solved, leading to a novel decentralized helper motion control scheme. Simulations indicate that introducing helper mobility leads to considerable savings in terms of helper transmit power, as well as total number of helpers required for secrecy communications.

Index Terms— Secrecy Rate, Cooperative Jamming, Physical Layer Security, Network Mobility Control, Cooperative Networks

1. INTRODUCTION

Information secrecy is a challenge in wireless communications, as the wireless channel makes the transmitted information accessible to unauthorized as well as legitimate nodes. Physical (PHY) layer secrecy exploits channel conditions to maximize the rate at which reliable information is delivered to the legitimate destination, with unauthorized users having the maximum possible uncertainty about the transmitted signal based on their observations [1]. In wireless communications, secrecy is typically addressed with cryptographic approaches, which rely on secret keys that are distributed to the network users periodically. However, breaking a secret key is only a matter of computational power and time. Although longer keys which are updated often are more difficult to be broken, however, they occupy valuable communication bandwidth. On the other hand, physical layer secrecy, exploits the

randomness of the wireless channel to maximize the ambiguity of the transmitted signal at an unauthorized receiver. Secrecy communication through broadcast channels was studied in [2] and for the scalar Gaussian wiretap channel in [3]. While for single-input single-output (SISO) channels perfect secrecy is achievable only when the Alice-Eve channel is a degraded version of the Alice-Bob channel [1, 3], the use of multiple antennas [4, 5] can ensure positive secrecy rate even when the SISO methods fail. Cooperative jamming (CJ) is another way to achieve high secrecy rates, and can be used when the wireless transceiver, due to size limitations, cannot accommodate more than one antennas. In CJ, helpers transmit noise to degrade the Alice-Eve channel. In most existing CJ schemes the helpers require full channel state information (CSI) on Eve [6, 7], which implicitly assumes that Eve is a known node in the network. Works that approach the problem requiring partial CSI on Eve, or no CSI at all, include [8, 9].

Mobility has been shown to dramatically improve the capacity of multiuser ad hoc wireless networks with random relay-assisted source-destination pairs [10]. In [11, 12], distributed network mobility control is employed in order to guide the network nodes for achieving specific operational goals, such as the adaptation of the network topology to a jamming attack, motivated by classical mechanics and through the use of carefully designed sequential algorithms. Also, more recently, in [13], mobility control has been combined with optimal beamforming for transmit power minimization under Quality of Service (QoS) constraints in multiuser cooperative networks.

This paper follows up on the recent work by some of the authors [9], in which, the communication between Alice and Bob is aided by multi-antenna helpers. Each helper transmits noise that is in the null-space of the helper's channel to Bob, thus creating interferences at all points in space, which include Eve, but not Bob. No coordination between helpers is required to design the noise, and no CSI on Alice is needed. By imposing nulling at Bob, the scheme of [9] is suboptimal, however, it was shown [9] to perform very close to a

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scheme that transmits optimally designed noise. By exploiting the dependence of jamming noise on the helper positions, we propose a method that further improves the secrecy rate performance. In particular, we consider the problem of jointly maximizing the system secrecy rate with respect to the helper weights and positions. It turns out that this joint optimization problem can be efficiently solved, leading to a novel decentralized helper motion control scheme. Numerical simulations show that the proposed scheme can lead to considerable savings in terms of helper transmitting power, as well as the total number of helpers required for meeting a certain secrecy rate.

Our work assumes that each helper has a map of its channel to Bob, Alice and Eve. Such map can be constructed based on the exchange of messages, which assumes that Eve is a known quantity in the network; for example, Eve is not an enemy, but rather a node that has limited access to information and certain communications need to be kept secret from her.

The rest of this paper is organized as follows. In Section 2, we present the system model. In Section 3, we briefly describe the nulling noise technique for stationary helpers, and in Section 4, we combine this technique with helper mobility control, leading to our proposed decentralized motion control scheme for joint maximization of the system secrecy rate. Finally, in Section 5, we present numerical simulations, illustrating the effectiveness of the proposed approach.

2. SYSTEM MODEL

The system model is depicted in Fig. 1. Alice wishes to communicate with Bob, keeping the transmitted information secret from Eve. Alice, Bob and Eve are each equipped with a single transmission/reception antenna and are located at positions $\mathbf{p}_i \in \mathbb{R}^2, i \in \{A, B, E\}$, respectively.

The transmission is assisted by R helpers, each of which is equipped with N_r antennas, with $N_r \geq 2, r \in \mathbb{N}_R^+ \equiv \{1, 2, \dots, R\}$. The area spanned by each helper is considered a disc with diameter ρ and center located at position $\mathbf{p}_r \in \mathbb{R}^2$. Without any ambiguity, we will refer to \mathbf{p}_r as the position of helper r . Moreover, the antennas of each helper r are located at positions $(\mathbf{p}_r + \mathbf{p}_r^{\rho k}) \in \mathbb{R}^2, k \in \mathbb{N}_{N_r}^+$, so that the spacing between any two antennas on the same helper is at least $\lambda/2$, where λ denotes the carrier wavelength used for the communication.

We assume that, throughout each transmission cycle, Alice, Bob and Eve are stationary, while the helpers can independently move at will. Based on this assumption, the channels from Alice to Bob and Eve are denoted by $h_A(\mathbf{p}_A, \mathbf{p}_B) \equiv h_A \in \mathbb{C}$ and $g_A(\mathbf{p}_A, \mathbf{p}_E) \equiv g_A \in \mathbb{C}$, respectively. Also, the channels from each helper to Bob and Eve are denoted by $\mathbf{h}_r(\mathbf{p}_B, \mathbf{p}_r) \equiv \mathbf{h}_r(\mathbf{p}_r) \in \mathbb{C}^{N_r}$ and $\mathbf{g}_r(\mathbf{p}_E, \mathbf{p}_r) \equiv \mathbf{g}_r(\mathbf{p}_r) \in \mathbb{C}^{N_r}, r \in \mathbb{N}_R^+$, respectively. Assuming that rotational motion is not allowed, then, for each helper, the channel gain for each specific antenna can

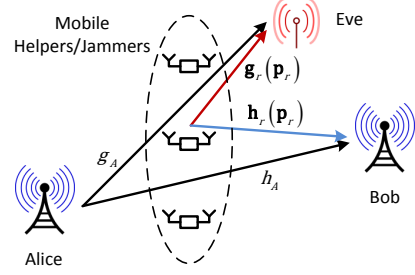


Fig. 1. System Model

be parametrized with respect to its position, since, if \mathbf{p}_r is determined, the positions of all its antennas can be deterministically set from its respective antenna topology. More specifically, we define

$$\mathbf{h}_r(\mathbf{p}_r) \triangleq [h_{r1}(\mathbf{p}_r + \mathbf{p}_r^{\rho 1}) \dots h_{rN_r}(\mathbf{p}_r + \mathbf{p}_r^{\rho N_r})]^T, \quad (1)$$

$$\mathbf{g}_r(\mathbf{p}_r) \triangleq [g_{r1}(\mathbf{p}_r + \mathbf{p}_r^{\rho 1}) \dots g_{rN_r}(\mathbf{p}_r + \mathbf{p}_r^{\rho N_r})]^T. \quad (2)$$

Alice transmits the signal $\sqrt{P_s}x$, where P_s denotes the transmission power and x is assumed to be an arbitrary zero mean complex random variable with $\mathbb{E}\{|x|^2\} = 1$. Additionally, we assign an individual power budget to each helper denoted as $P_r, r \in \mathbb{N}_R^+$.

We consider a mobility - enabled cooperative jamming scenario, that is, while Alice is transmitting, all helpers can independently move and independently transmit the artificial noise signal $\mathbf{n}_r(\mathbf{p}_r) \in \mathbb{C}^{N_r}, r \in \mathbb{N}_R^+$, which is also independent of the information transmitted. Under this setting, Bob and Eve receive the signals

$$y_B \triangleq \sqrt{P_s}h_A x + \sum_{r=1}^R \mathbf{h}_r^T(\mathbf{p}_r) \mathbf{n}_r(\mathbf{p}_r) + n_B, \quad (3)$$

$$y_E \triangleq \sqrt{P_s}g_A x + \sum_{r=1}^R \mathbf{g}_r^T(\mathbf{p}_r) \mathbf{n}_r(\mathbf{p}_r) + n_E, \quad (4)$$

respectively, where n_B and n_E denote complex Additive White Gaussian Noise (AWGN) quantities at the respective reception points with equal variances $\mathbb{E}\{|n_B|^2\} = \mathbb{E}\{|n_E|^2\} = N_0$.

In the following, we assume that the quantities $\sqrt{P_s}, h_A, g_A, \mathbf{h}_r(\mathbf{p}_r), \mathbf{g}_r(\mathbf{p}_r)$ are known. For each position of the helpers in the environment, our goal is to design the jamming noise $\mathbf{n}_r(\mathbf{p}_r)$ and at the same time control the positions of the helpers $\mathbf{p}_r, r \in \mathbb{N}_R^+$ so that the secrecy rate of the whole system is maximized.

The wireless channel is assumed to be flat fading. Assuming a rich scattering environment [13], the baseband equivalent channel gain $c_{ij}(\mathbf{p}_i, \mathbf{p}_j) \in \mathbb{C}$ between two arbitrary antennas i and j with respective positions \mathbf{p}_i and \mathbf{p}_j and Euclidean distance $d_{ij}(\mathbf{p}_i, \mathbf{p}_j) \equiv d_{ij}$, can be approximated as $c_{ij}(\mathbf{p}_i, \mathbf{p}_j) = \alpha_{ij}(\mathbf{p}_i, \mathbf{p}_j) \beta_{ij} e^{j(2\pi/\lambda)d_{ij}}$, where

$\alpha_{ij}(\mathbf{p}_i, \mathbf{p}_j) \sim \mathcal{CN}(0, 1/2)$ models multipath fading, and $\beta_{ij} = d_{ij}^{-\mu/2}$ models path loss, where μ denotes the path loss coefficient and represents the power fall-off rate of the channel. Further, we assume that $\alpha_{ij}(\mathbf{p}_i, \mathbf{p}_j)$ and $\alpha_{kj}(\mathbf{p}_k, \mathbf{p}_j)$ are independent if $d_{ik} \geq \lambda/2$ and correlated otherwise. Thus, for a fixed position of, for instance, an antenna j , we can create a spatial map that returns the multipath fading coefficient $\alpha_{ij}(\mathbf{p}_i, \mathbf{p}_j)$ (and of course $c_{ij}(\mathbf{p}_i, \mathbf{p}_j)$) for any position of the antenna i in the environment.

3. GENERATION OF NULLING NOISE

Each helper generates Nulling Noise (NN), so that Bob is interference-free [9]. More specifically, for fixed positions throughout the plane, the noise transmitted by each helper is designed so that

$$\mathbf{h}_r^T(\mathbf{p}_r) \mathbf{n}_r(\mathbf{p}_r) = 0, \quad r \in \mathbb{N}_R^+ \quad (5)$$

For $N_r \geq 2$, $r \in \mathbb{N}_R^+$, (5) enjoys the simple closed form solution $\mathbf{n}_r(\mathbf{p}_r) = \mathbf{E}_r(\mathbf{p}_r) \mathbf{v}_r$, where $\mathbf{E}_r(\mathbf{p}_r) \in \mathbb{C}^{N_r \times (N_r-1)}$ denotes a column-orthonormal basis matrix for the right nullspace of $\mathbf{h}_r^T(\mathbf{p}_r)$ and $\mathbf{v}_r \in \mathbb{C}^{(N_r-1) \times 1}$ denotes an arbitrary vector. Of course, $\mathbf{E}_r(\mathbf{p}_r)$ can be easily obtained from $\mathbf{h}_r^T(\mathbf{p}_r)$ through a usual singular value decomposition. By setting $\mathbf{v}_r \triangleq w_r \mathbf{t}_r$, where $w_r \in \mathbb{C}$ and $\mathbf{t}_r \sim \mathcal{CN}(0, \mathbf{I}_{(N_r-1) \times (N_r-1)})$, we can design the jamming noise to be of the form [9]

$$\mathbf{n}_r(\mathbf{p}_r) = w_r \mathbf{E}_r(\mathbf{p}_r) \mathbf{t}_r. \quad (6)$$

Then, the power of the jamming noise is given by

$$\begin{aligned} \mathbb{E} \left\{ \|\mathbf{n}_r(\mathbf{p}_r)\|^2 \right\} &= \mathbb{E} \left\{ \text{Tr}(\mathbf{n}_r(\mathbf{p}_r) \mathbf{n}_r^H(\mathbf{p}_r)) \right\} \\ &= |w_r|^2 \text{Tr}(\mathbf{E}_r(\mathbf{p}_r) \mathbf{E}_r^H(\mathbf{p}_r)) = (N_r - 1) |w_r|^2 \leq P_r. \end{aligned}$$

Note that the noise power is independent of the position of each helper.

In Section 4, we combine the NN technique with helper mobility control so that the secrecy rate of the system defined in Section 2 is maximized.

4. JOINT DECENTRALIZED MOBILITY CONTROL AND NULLING NOISE FOR SECRECY RATE MAXIMIZATION

Using the NN approach, the secrecy rate of the system can be expressed as [9]

$$\begin{aligned} R(\mathbf{w}, \mathbf{p}) &= \log_2 \left(1 + \frac{P_s |h_A|^2}{N_0} \right) \\ &\quad - \log_2 \left(1 + \frac{P_s |g_A|^2}{\sum_{r=1}^R |w_r|^2 \phi_r(\mathbf{p}_r) + N_0} \right), \quad (7) \end{aligned}$$

where

$$\mathbf{w} \triangleq [|w_1|^2, \dots, |w_R|^2]^T \in \mathbb{R}^R, \quad (8)$$

$$\mathbf{p} \triangleq [\mathbf{p}_1^T, \dots, \mathbf{p}_R^T]^T \in \mathbb{R}^{2R} \quad \text{and} \quad (9)$$

$$\phi_r(\mathbf{p}_r) \triangleq \|\mathbf{E}_r^H(\mathbf{p}_r) \mathbf{g}_r(\mathbf{p}_r)\|^2 \in \mathbb{R}^+, \quad r \in \mathbb{N}_R^+. \quad (10)$$

We are interested in the joint maximization of the system secrecy rate with respect to the weight vector \mathbf{w} and the positions vector \mathbf{p} , while satisfying the power budget constraints $P_r, r \in \mathbb{N}_R^+$ for each helper, that is, we are interested in the optimization problem

$$\begin{aligned} \max_{\mathbf{w}, \mathbf{p}} \quad & R(\mathbf{w}, \mathbf{p}) \\ \text{s.t.} \quad & |w_r|^2 \leq \frac{P_r}{N_r - 1}, \quad r \in \mathbb{N}_R^+. \end{aligned} \quad (11)$$

Obviously, $R(\mathbf{w}, \mathbf{p})$ is bounded from above by the first term on the right-hand side of (7), where that bound is clearly not attainable for any admissible choice of \mathbf{w} and/or \mathbf{p} and thus constitutes a reference supremum for (11).

By definition of the system secrecy rate in (7), the optimization problem (11) is equivalent to

$$\begin{aligned} \max_{\mathbf{w}} \max_{\mathbf{p}} \quad & \sum_{r=1}^R |w_r|^2 \phi_r(\mathbf{p}_r) \\ \text{s.t.} \quad & |w_r|^2 \leq \frac{P_r}{N_r - 1}, \quad r \in \mathbb{N}_R^+ \end{aligned} \quad (12)$$

Obviously, at the optimal solution of (12) with respect to \mathbf{w} , all the constraints will be active and as a result, (12) is equivalent to

$$\max_{\mathbf{p}} \quad \sum_{r=1}^R \beta_r \phi_r(\mathbf{p}_r), \quad (13)$$

which is a simple unconstrained optimization problem and where $\beta_r \triangleq P_r / (N_r - 1)$, $r \in \mathbb{N}_R^+$. We see that regardless of its position, helper r should use its maximum power budget P_r for noise transmission.

Now, we will additionally assume that each helper reacts to the continuous time control input $\mathbf{u}_r \in \mathbb{R}^2$, according to the first order differential equation

$$\dot{\mathbf{p}}_r = \mathbf{u}_r, \quad r \in \mathbb{N}_R^+. \quad (14)$$

Under this assumption, our goal is to determine the motion controllers $\mathbf{u}_r, \forall r \in \mathbb{N}_R^+$ that ensure maximization of the secrecy rate of the system $R(\mathbf{w}, \mathbf{p})$, or equivalently the solution of the optimization problem (13).

Similar to [13], by introducing an additional collision constraint among each helper and all the other nodes in the network, and by noting that $\beta_r > 0, \forall r \in \mathbb{N}_R^+$, we can obtain the desired controllers $\mathbf{u}_r, r \in \mathbb{N}_R^+$ as the negative gradient

of appropriately chosen artificial potential functions for each helper $\phi_r^o : \mathbb{R}^2 \rightarrow \mathbb{R}^+$, $r \in \mathbb{N}_R^+$, defined as

$$\phi_r^o(\mathbf{p}_r) = \phi_r^{col}(\mathbf{p}_r) - \phi_r(\mathbf{p}_r). \quad (15)$$

In (15), $\phi_r(\mathbf{p}_r)$ is given by (10) and

$$\phi_r^{col}(\mathbf{p}_r) \triangleq \sum_{l \in \mathcal{S}} \frac{1}{\|\mathbf{p}_r - \mathbf{p}_l\|^2 - \rho^2}, \quad (16)$$

where $\mathcal{S} \triangleq \{\mathbb{N}_R^+, A, B, E\} - \{r\}$.

After defining the potentials $\phi_r^o(\mathbf{p}_r)$, $r \in \mathbb{N}_R^+$, the desired helper controllers are obtained as

$$\mathbf{u}_r = -\nabla_{\mathbf{p}_r} \phi_r^o(\mathbf{p}_r), \quad \forall r \in \mathbb{N}_R^+, \quad (17)$$

leading to the set of closed loop systems

$$\dot{\mathbf{p}}_r = -\nabla_{\mathbf{p}_r} \phi_r^o(\mathbf{p}_r), \quad r \in \mathbb{N}_R^+. \quad (18)$$

It is evident that the gradient $\nabla_{\mathbf{p}_r} \phi_r^o(\mathbf{p}_r)$ can be computed independently by the respective helper, using only local information, since the quantities $\mathbf{E}_r^H(\mathbf{p}_r)$ and $\mathbf{g}_r(\mathbf{p}_r)$ that appear in $\phi_r(\mathbf{p}_r)$ depend solely on the channels from helper r to Bob and Eve, respectively. This means that (18) constitutes a purely decentralized scheme for controlling independently each helper, leading to the maximization of the system secrecy rate. However, we should note that since the potentials $\phi_r^o(\mathbf{p}_r)$, $r \in \mathbb{N}_R^+$ are generally non convex functions with respect to \mathbf{p}_r , the proposed control scheme can only guarantee local optimality and this mostly depends on the considered approach for wireless channel modeling.

5. EXPERIMENTAL RESULTS

Although the control scheme proposed in Section 4 is only locally optimal with respect to the positions of the helpers, we present here numerical simulations, showing that our approach can lead to considerable savings in terms of both the helper transmitting power and also the total number of helpers needed for high secrecy rate communication.

We consider a wireless network in a $(3 \times 5) m$ rectangular plane, with of Alice, Bob and Eve at points $(1.5, 0.1)$, $(1.5, 4.9)$, $(1.5, 4.1)$, respectively. We also assume that the network is assisted by several 2-antenna helpers performing cooperative jamming with NN, initially placed with equal spacing across dimension x and randomly across dimension y with ordinates $2.5 + \epsilon$, where ϵ is a random variable, with $\epsilon \sim \mathcal{U}[-\gamma, \gamma]$, $0 < \gamma \ll 1$. This choice for the initial arrangement of the helpers in the plane is justified by the fact that in order to be able to fairly compare the performance of the system for different numbers of helpers, the helpers should sense similar channel magnitudes with respect to Bob and Eve. This is due to path loss. Also, regarding channel modeling, we assume that $\mu = 3.5$, a reasonable value for mobile robotics applications, and $\lambda = 0.4 m$. Further, Bob's Signal to Noise Ratio (SNR) is fixed at 20 dB and the Jamming Noise to Noise Ratio (JNNR) for all helpers is fixed at 17 dB.

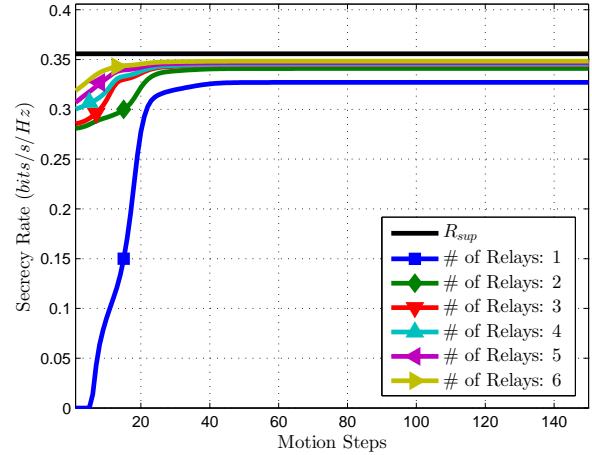


Fig. 2. Secrecy rate evolution for various numbers of helpers.

Fig. 2 shows comparatively the evolution of the secrecy rate, when the number of the helpers varies from 1 to 6, as well as the upper bound of the system secrecy rate R_{sup} , for 150 motion steps of the proposed control scheme. In the case where the network is assisted by only 1 helper, we observe that whereas the initial value of the secrecy rate is 0, after 150 motion steps, it eventually reaches a value very close to its supremum. On the other hand, when helper mobility is not employed, from Fig. 2 (rates corresponding to 1 motion step) we can see that in order for a comparable secrecy rate to be achieved, the number of helpers must be at least 6. Therefore, using the proposed approach, the secrecy rate can be maximized using a much smaller number of helpers, resulting in large savings in terms of the physical resources required for a secure, high - secrecy rate system.

Additionally, our approach not only requires fewer helpers, but also results in smaller helper transmitting power requirements. This fact can be justified by observing that if, for our example, the total available number of helpers was constrained to, say, 2, then, in order to achieve a secrecy rate close to the R_{sup} without mobility control, we would have to increase the JNNR (power budgets) of the helpers.

6. CONCLUSIONS

In this paper, we have addressed the problem of improving the secrecy rate in single-source single-destination networks by employing mobile jammers. We have proposed a novel decentralized motion control scheme for the helpers that effectively maximizes the secrecy rate of the system. Finally, through numerical simulations, we have showed that the proposed scheme can yield considerable savings in terms of both physical and power network resources. This makes our approach very promising for further research and application in cooperative wireless networks.

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