Title: Connectivity of Dynamic Graphs

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Abstract

Wireless communication is known to play a pivotal role in enabling teams of robots to successfully accomplish global coordinated tasks. In fact, network connectivity is an underlying assumption in every distributed control and optimization algorithm. For this reason, in recent years, there is growing research in designing controllers that ensure point-to-point or end-to-end network connectivity for all time. Such controllers either rely on graph theory to model robot communication or employ more realistic communication models that take into account path loss, shadowing, and multi-path fading. Nevertheless, maintaining all-time connectivity can severely restrict the robots

from accomplishing their tasks, as motion planning is always constrained by network connectivity constraints. Therefore, intermittent connectivity controllers have recently been proposed, as well, that allow the robots to communicate in an intermittent fashion and operate in disconnect mode the rest of the time. This article provides an overview of all-time and intermittent connectivity controllers.

Connectivity Control of Multi-Robot Systems

Graph-theoretic Connectivity Control

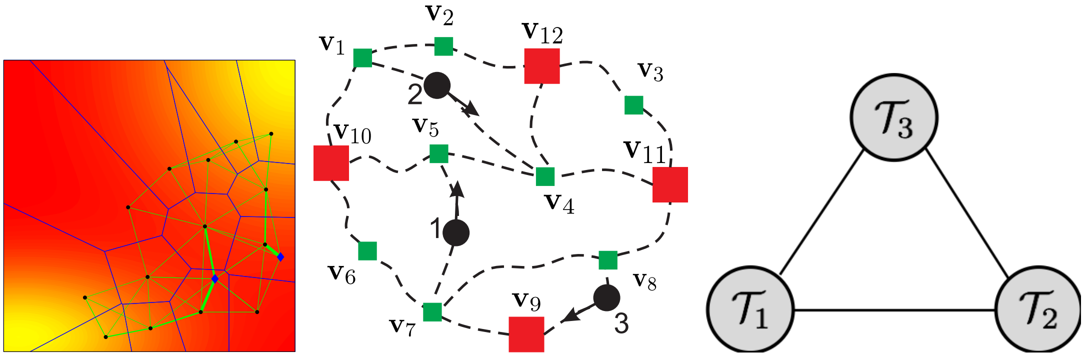
Recent methods for communication control of mobile robot networks typically rely on proximity graphs to model the exchange of information among the robots and, therefore, the communication problem becomes equivalent to preserving graph connectivity. Methods to control graph connectivity typically rely on controlling the Fiedler value of the underlying graph. One possible way of doing so is maximizing the Fiedler value in a centralized [Kim and Mesbahi(2006)] or distributed [DeGennaro and Jadbabaie(2006)] fashion. Alternatively, potential fields that model loss of connectivity as an obstacle in the free space can be employed for this purpose, as shown in [Zavlanos and Pappas(2007)]. A distributed hybrid approach to connectivity control is presented in [Zavlanos and Pappas(2008)] whereby communication links are efficiently manipulated by decoupling the continuous robot motion from the control of the discrete graph. Further distributed algorithms for graph connectivity maintenance have been implemented in [Notarstefano et al(June 2006)Notarstefano, Savla, Bullo, and Jadbabaie, Ji and Egerstedt(August 2007), Sabattini et al(2013)Sabattini, Chopra, and Secchi]. A comprehensive survey of this literature can be found in [Mesbahi and Egerstedt(2010), Zavlanos et al(2011)Zavlanos, Egerstedt, and Pappas].

Connectivity Control using Realistic Communication Models

In practice, the above graph-based communication models turn out to be rather conservative, since proximity does not necessarily imply tangible and reliable communication. More realistic communication models for mobile networks compared to the above graph-theoretic models are proposed in [Zavlanos et al(2010)Zavlanos, Ribeiro, and Pappas, Zavlanos et al(2013)Zavlanos, Ribeiro, and Pappas, Stephan et al(2017)Stephan, Fink, Kumar, and Ribeiro] that take into account the routing of packets as well as desired bounds on the transmitted rates. The key idea in these works is to define connectivity in terms of communication rates and to use optimization methods to determine optimal operating points of wireless networks. A conceptually similar communication model is proposed in [Fink et al(November 2010)Fink, Ribeiro, Kumar, and Sadler]. Integration of these communication frameworks with area coverage control and navigation tasks in complex environments is presented in [Kantaros and Zavlanos(2016b), Kantaros and Zavlanos(2016a)]; see also Figure 1(a). A communication model that accounts for multi-path fading of channels is proposed in [Lindhé and Johansson(2010)], where robot mobility is exploited in order to increase the throughput. Multi-path fading, shadow fading, and path loss have also been used to model channels in [Mostofi et al(2010)Mostofi, Malmirchegini, and Ghaffarkhah, Malmirchegini and Mostofi(2012)]. In these works, a probabilistic framework for channel prediction is developed based on a small number of measurements of the received signal power. The integration of the latter communication models with robot mobility is described in [Ghaffarkhah and Mostofi(2010)].

Intermittent Connectivity Control

Common in the above works is that point-to-point or end-to-end network connectivity is required to be preserved for all time. However, due to the uncertainty in the wireless



(a) Area Coverage (b) Intermittent Connectivity (c) Team Membership Graph

Fig. 1. Figure 1(a) shows a communication network consisting of 14 robots (black dots) that are tasked with optimally sensing two sources, whose density is captured in yellow, while reliably transmitting the collected information to 2 access points (blue rhombuses). The thickness of communication links (green edges) depends on their reliability. Figure 1(b) illustrates a joint task planning and intermittent connectivity framework. The robots have to accomplish complex tasks, such as data gathering, by visiting the green waypoints, and exchange the collected information by communicating intermittently when they simultaneously visit the communication points (red squares). The teams are defined as $\mathcal{T}_1 = \{1, 2\}$, $\mathcal{T}_2 = \{2, 3\}$, and $\mathcal{T}_3 = \{1, 3\}$. The corresponding team membership graph is shown in Figure 1(c).

channel, it is often impossible to ensure all-time connectivity in practice. Moreover, all-time connectivity constraints may prevent the robots from moving freely in their environment to fulfill their tasks, and instead favor motions that maintain a reliable communication network. Motivated by this fact, intermittent communication frameworks have recently been proposed that allow the robots to communicate intermittently and operate in disconnect mode the rest of the time. Specifically, [Hollinger and Singh(2010), Hollinger et al(2012)Hollinger, Singh et al] propose a receding horizon framework for periodic connectivity that ensures recovery of connectivity within a given time horizon. A similar recurrent connectivity framework for planning on environments modeled as graphs is proposed in [Banfi et al(2018)Banfi, Basilico, and Amigoni]. Unlike these approaches that require the whole network to become occasionally re-connected, alternative approaches have been recently proposed that do not require the whole communication network to ever become connected at once, but they ensure connectivity over time, infinitely often [Zavlanos(2010)]. The key idea is to divide the robots into $M > 0$ smaller teams $\{\mathcal{T}_m\}_{m=1}^{M}$, where M is user-specified, and require that communication events take place when the robots in every team meet at a common location in space that can be fixed or optimally selected; see also Figure 1(b). While in disconnect mode, the robots can accomplish other tasks free of communication constraints. The teams $\mathcal{T}_m$ are constructed so that every robot belongs to at least one team giving rise to a team membership graph $\mathcal{G}_{\mathcal{T}} = (\mathcal{V}_{\mathcal{T}}, \mathcal{E}_{\mathcal{T}})$, where the set of nodes $\mathcal{V}_{\mathcal{T}}$ is indexed by the teams $\mathcal{T}_m$ and set of edges $\mathcal{E}_{\mathcal{T}}$ is defined as $\mathcal{E}_{\mathcal{T}} = \{(m,n) | \mathcal{T}_m \cap \mathcal{T}_n \neq \emptyset, \forall m,n \in \mathcal{V}_{\mathcal{T}}, m \neq n\}$. Assuming that the team membership graph $\mathcal{G}_{\mathcal{T}}$ is a connected bipartite graph, [Zavlanos(2010)] proposes a distributed control scheme that achieves periodic communication events, synchronously, at meeting/communication locations. Arbitrary connected team graphs are considered in [Kantaros and Zavlanos(July 2017), Kantaros and Zavlanos(2016c)] where techniques from formal methods are employed to synthesize correct-by-construction discrete plans (schedules) that determine the order in which communication events should occur at the meeting locations. Specifically, in these works, intermittent connectivity is captured by the following global Linear Temporal Logic (LTL) formula:

$$\phi = \wedge_{m\in\mathcal{M}} \left(\square\lozenge \left(\vee_{j\in\mathcal{C}_m} (\wedge_{i\in\mathcal{T}_m} \pi_i^{\mathbf{v}_j})\right)\right), \tag{1}$$

that requires all robots in a team $\mathcal{T}_m$ to meet infinitely often at a common communication point, denoted by $\mathbf{v}_j$, for all teams $\mathcal{T}_m$. In (1), $\pi_i^{\mathbf{v}_j}$ is a Boolean variable that is true if robot i is in location $\mathbf{v}_j$, and $\mathcal{C}_m$ is a set that collects all communication points $\mathbf{v}_j$ available to team $\mathcal{T}_m$. Also, $\square\lozenge$ represents the infinitely often requirement. A detailed presentation of LTL semantics can be found in [Baier and Katoen(2008)]. Then, [Kantaros and Zavlanos(July 2017), Kantaros and Zavlanos(2016c)] propose a distributed control synthesis algorithm to synthesize schedules that satisfy ϕ. Integration of this intermittent connectivity framework with temporal logic task planning, state estimation tasks, and planning for time-critical and dynamic tasks is proposed in [Kantaros et al(2019)Kantaros, Guo, and Zavlanos], [Khodayi-mehr et al(2019)Khodayi-mehr, Kantaros, and Zavlanos], and [Kantaros and Zavlanos(May 2018)], respectively. An intermittent communication framework for star-like communication topologies is also presented in [Guo and Zavlanos(2018)] that considers information flow only to the root/user and not among the robots.

Summary and Future Research Directions

Network connectivity is an underlying assumption in every distributed control and optimization algorithm. As a result, all-time connectivity controllers that either rely on graph theory to model robot communication or employ more realistic communication models have been proposed to control communication between nodes. When all-time communication is impossible or very restrictive of a condition, intermittent connectivity controllers can be used that allow the robots to communicate in an intermittent fashion and operate in disconnect mode the rest of the time. Intermittent connectivity control in unknown and dynamic environments, such as crowded human environments or underwater environments and resilient intermittent connectivity controllers that are e.g., robust to robot failures are interesting future research directions.

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