# Single-Agent Indirect Herding of Multiple Targets using Metric Temporal Logic Switching

Duc M. Le[1], Xusheng Luo[2], Leila J. Bridgeman[2], Michael M. Zavlanos[2], Warren E. Dixon[1]

***Abstract*—This paper focuses on the single-agent indirect herding problem where a herder agent is tasked with regulating a group of target agents to a desired goal state. To achieve this goal, the herder must switch between controlling different targets, resulting in a switched dynamical system. Lyapunov-based switched system analysis methods are used to develop sufficient dwell-time conditions. These dwell-time conditions are then encoded into Metric Temporal Logic (MTL) specifications that constitute the constraints in a switched nonlinear Model Predictive Control (MPC) problem that is formulated to synthesize a desired switching protocol. The synthesized switching protocol ensures the herding objective is achieved while simultaneously satisfying the dwell-time MTL specifications.**

## I. Introduction

Networked control systems typically consist of collections of agents that collaborate to accomplish global objectives such as consensus, synchronization, and formation control [1]. These tasks require the agents to comply with explicit control protocols while possibly respecting desired communication constraints. Nevertheless, in many practical applications, agents may lack navigational sensing capabilities or may be non-cooperative, which makes their control particularly challenging. In such scenarios, it can be beneficial to seek help from other agents in the network. Herding enables one (or multiple) agents to indirectly control other agents, despite potentially lacking knowledge of or having different control objectives compared to the herding agent(s). Herding typically focuses on regulating the states of a group of agents to a desired goal state. In the literature, foundational approaches to the herding problem were biologically inspired and relied on repulsive interaction dynamics between the herders and targets to achieve the herding objective [2]–[5]. More recently, multi-agent herding problems have also been studied that adopt circular arc formation control methods to drive the targets to their goals [6], while also considering adversarial agents and obstacle avoidance [7]. In the aforementioned results, herding strategies either require multiple herder agents or assume a cohesive behavior among target agents.

[1]Duc M. Le and Warren E. Dixon are with the Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville FL 32611-6250, USA. (e-mail: {ledan50, wdixon}@ufl.edu)

[2]Xusheng Luo, Leila J. Bridgeman, and Michael M. Zavlanos are with the Department of Mechanical Engineering and Materials Science, Duke University, Durham, NC 27708, USA. (e-mail: {xusheng.luo, leila.bridgeman, michael.zavlanos}@duke.edu)

This research is supported in part by AFOSR award number FA9550-19-1-0169, NEEC award number N00174-18-1-0003, and ONR under agreement number N00014-18-1-2374. Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the sponsoring agency.

This paper focuses on single-agent indirect herding, introduced in [8]. Specifically, a single herder agent achieves the herding objective through interactions with non-cooperative target agents. Interactions between the herder and the target agents are modeled by nonlinear potential functions that represent a repulsive effect based on the distance between the agents. Similar to [8]–[10], it is assumed that the herder agent is limited to regulating one target agent at a time. While a target agent is regulated to a goal state by the herder, the remaining target agents can potentially diverge from their goal states. To ensure all targets eventually reach a desired goal state, a switching strategy is developed to enable the herder agent to intermittently interact with each target agent. Such a switching strategy results in a system that cyclically switches between two modes—a stabilizable mode and an uncontrolled unstable mode. To analyze stability of the switched system, Lyapunov-based methods are used to develop sufficient dwell-time conditions, as in [11] and [12]. Although dwell-time conditions provide stability criteria for allowable switching behaviors, they are only sufficient and lack constructive methods to design valid switching protocols. As a result, switching protocols are often designed to satisfy dwell-time conditions in a brute force manner, lacking the capability to exhibit optimal performance in the system.

Unlike [8]–[10], this paper proposes a Metric Temporal Logic (MTL) approach to construct a switching protocol. Specifically, the dwell-time conditions are encoded into MTL specifications and then a switched nonlinear Model Predictive Control (MPC) problem is formulated. Switched nonlinear MPC methods are considered in [13]–[15], but do not consider MTL specifications as constraints to the MPC. Hence, we develop a method to synthesize a switching protocol that satisfies the herding objective, the nonlinear agent dynamics, and the dwell-time MTL specifications. Additionally, stability of the switched MPC is provided.

MTL was first introduced in [16] as a high-level formalism to capture time-constrained specifications used in verification of system behavior. Specifications expressed for MTL have been incorporated into controlled systems, such as robot manipulators [17], [18], quad-rotors [19], mobile robots [20]–[22], and vehicle routing [23]. Common approaches to synthesize controllers under MTL specifications include automata-based methods [24] and optimization-based methods. This paper adopts the latter approach and treat the MTL specifications as constraints to an optimal control problem [25]. A similar MTL-based approach was also investigated in [26] to synthesize a controller for consensus in a leader-follower network, but for a class of linear switched systems. Compared to [26], this paper focuses on the indirect herding problem for a class of nonlinear switched systems.

## II. Preliminaries

This section provides the syntax and semantics of MTL. MTL is a type of formal logic which is defined over a set of atomic propositions $\mathcal{AP}$, Boolean operators, conjunction $\wedge$ and negation $\neg$, and temporal operators "until" $\mathcal{U}$. The syntax of MTL follows the grammar

$$\phi ::= \top \,|\, \pi \,|\, \phi_1 \wedge \phi_2 \,|\, \phi_1 \vee \phi_2 \,|\, \neg\phi \,|\, \phi_1 \mathcal{U}_{\mathcal{I}}\, \phi_2,$$

where $\top$ is unconditionally true, $\pi$ is an atomic proposition that represents a statement evaluated as either true or false at any time, $\phi_1$ and $\phi_2$ are MTL formulas, and $\mathcal{I}$ is a time interval of the form $\mathcal{I} = [a, b]$ such that $a, b \in \mathbb{N}_{\geq 0}$ and $a \leq b$. The two operators "eventually" $\Diamond$ and "always" $\Box$ can be derived from the operator "until" $\mathcal{U}$ as $\Diamond_{\mathcal{I}}\phi = \top \mathcal{U}_{\mathcal{I}}\, \phi$ and $\Box_{\mathcal{I}}\phi = \neg\Diamond_{\mathcal{I}}\neg\phi$, respectively.

Let $\{b(0)b(1)b(2)\ldots b(t_j)\ldots\}$ be a discrete-timed sequence of binary variables such that $b(t_j)$ is true or false for all time indices $t_j \in \mathbb{N}_{\geq 0}$.[1] The semantics of an MTL formula, which is defined later, is evaluated over such timed sequences. The sequence should be sufficiently long to definitively determine whether it satisfies the MTL formula. This shortest length is referred to as the necessary length of an MTL formula $\phi$, denoted by $\|\phi\|$, which is recursively defined as [27]

$$\begin{aligned} \|\pi\| &= 0 \\ \|\neg\phi\| &= \|\phi\| \\ \|\phi_1 \wedge \phi_2\| &= \max(\|\phi_1\|, \|\phi_2\|) \\ \|\phi_1 \mathcal{U}_{[a,b]}\, \phi_2\| &= b + \max(\|\phi_1\|, \|\phi_2\|). \end{aligned}$$

However, satisfaction of formulas such as $\Box_{[0,\infty]}\pi$ require infinite sequences. To enable a satisfactory evaluation over a discrete-time sequence of finite length, we distinguish between the strong and weak satisfaction relations. Specifically, let $b(0:h) = b(0)b(1)\ldots b(h)$, where the integer $h \in \mathbb{N}_{\geq 0}$ is positive. Then, the finite sequence $b(0:h)$ is recursively defined to strongly or weakly satisfy the formula $\phi$ at time index $t_j$, denoted by $b(0:h), t_j \models_S \phi$ and $b(0:h), t_j \models_W \phi$, respectively. Moreover, the sequence $b(0:h)$ strongly or weakly satisfies $\phi$ if it strongly or weakly satisfies $\phi$ at time 0, i.e., $b(0:h), 0 \models_S \phi$ or $b(0:h), 0 \models_W \phi$, respectively.

**Definition 1.** A discrete-time sequence $b(0:h)$ of finite length strongly satisfies the MTL formula $\phi$ at time index $t_j$ if [26]:

$$\begin{aligned} b(0:h), t_j \models_S \top \;&\Leftrightarrow\; t_j \leq h \;\wedge\; b(t_j) = \top, \\ b(0:h), t_j \models_S \neg\phi \;&\Leftrightarrow\; b(0:h), t_j \not\models_W \phi \\ b(0:h), t_j \models_S \phi_1 \wedge \phi_2 \;&\Leftrightarrow\; b(0:h), t_j \models_S \phi_1 \\ &\qquad \wedge\; b(0:h), t_j \models_S \phi_2, \\ b(0:h), t_j \models_S \phi_1 \mathcal{U}_{\mathcal{I}}\, \phi_2 \;&\Leftrightarrow\; \exists t_j' \in t_j + \mathcal{I}, s.t. \\ b(0:h), t_j' \models_S \phi_2 \wedge b(0:h)&, t_j'' \models_S \phi_1, \forall t_j'' \in [t_j, t_j'). \end{aligned}$$

[1]For simplicity, we restrict to the sequence of binary variables, which corresponds to the sequence of switching signals that are subsequently defined in Section IV.

**Definition 2.** A discrete-time sequence $b(0:h)$ of finite length weakly satisfies the MTL formula $\phi$ at time index $t_j$ if [26]:

$$\begin{aligned} b(0:h), t_j \models_W \top \;&\Leftrightarrow\; (t_j \leq h \;\wedge\; b(t_j) = \top) \\ &\qquad \vee\; t_j > h, \\ b(0:h), t_j \models_W \neg\phi \;&\Leftrightarrow\; b(0:h), t_j \not\models_S \phi \\ b(0:h), t_j \models_W \phi_1 \wedge \phi_2 \;&\Leftrightarrow\; b(0:h), t_j \models_W \phi_1 \\ &\qquad \wedge\; b(0:h), t_j \models_W \phi_2, \\ b(0:h), t_j \models_W \phi_1 \mathcal{U}_{\mathcal{I}}\, \phi_2 \;&\Leftrightarrow\; \exists t_j' \in t_j + \mathcal{I}, s.t. \\ b(0:h), t_j' \models_W \phi_2 \wedge b(0:h)&, t_j'' \models_S \phi_1, \forall t_j'' \in [t_j, t_j'). \end{aligned}$$

Informally speaking, a finite timed sequence weakly satisfies the formula $\phi$ if it does not strongly violate it for now, and there are possibilities to extend the sequence to strongly satisfy the formula if its necessary length is finite.

## III. Problem Definition

Consider a network of $N \in \mathbb{N}_{\geq 3}$ agents composed of a single herder agent and multiple target agents. The states of the target agents are denoted by $x_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ for all $i \in T = \{1, 2, ..., N-1\}$ and their goal states are denoted by $x_{i,g} \in \mathbb{R}^n$ for all $i \in T$. It is assumed the goal locations are unknown to the targets and only known to the herder. Moreover, it is assumed that the targets can possibly be non-cooperative, i.e., the target agents may diverge from the goal locations. The state of the herder agent is denoted by $x_N : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$.

The dynamics of the target agents are modeled as

$$\dot{x}_i(t) = \alpha_i(\|x_i - x_N\|)(x_i - x_N) + f_i(x_i, t), \qquad (1)$$

where $f_i : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ for all $i \in T$ denote the drift dynamics, and $\alpha_i : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$ for all $i \in T$ denote nonlinear potential functions. In (1), the drift dynamics model the tendency of the target agents to diverge from the goal location, and the potential functions model the tendency of the target agent to be repelled (or attracted) by the herder agent. The potential functions are known for all target agents and depend on the distance between the herder and each target. The following assumptions are made on the target agent dynamics.

**Assumption 1.** The drift dynamics $f_i$ are locally Lipschitz and can be bounded by $\|f_i(x_i, t)\| \leq \overline{f_i}$ for all $x_i \in \mathbb{R}^n$ and $t \in \mathbb{R}_{\geq 0}$, where the positive constants $\overline{f}_i \in \mathbb{R}_{>0}$ are known for all $i \in T$. ■

**Assumption 2.** The potential functions $\alpha_i$ are locally Lipschitz and can be bounded by $\underline{\alpha}_i \leq \alpha_i(\|x_i - x_N\|) \leq \overline{\alpha}_i$ for all $x_i, x_N \in \mathbb{R}^n$, where the positive constants $\underline{\alpha}_i, \overline{\alpha}_i \in \mathbb{R}_{>0}$ are known for all $i \in T$. ■

For simplicity, the herder agent is modeled by the single integrator dynamics[2]

$$\dot{x}_N(t) = u(\sigma_i(t), t), \qquad (2)$$

where $u : \{0,1\}^{(N-1)} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ denotes the switching control input of the herder agent, and $\sigma_i : \mathbb{R}_{\geq 0} \to \{0,1\}$ for all

[2]Various dynamic models can also be used for the herder, where established methods could be applied to yield the desired behavior.

$i \in T$ denotes the switching signal for each target agent. The switching signal assigns targets to the herder, one at a time, to capture the fact that the herder agent can only regulate one target at a time. Thus, it is required to intermittently switch between different targets to drive all of them to their desired goals. In this paper, we consider the following problem.

**Problem 1.** Consider a group of $N-1$ target agents and a single herder agent satisfying the dynamics in (1) and (2), respectively. Determine a control input $u\left(\sigma_i\left(t\right),t\right)$ for the herder and corresponding switching signals $\sigma_i\left(t\right)$ for all target agents such that all targets are driven to their goal states.

To solve Problem 1, a Lyapunov-based switched system approach is used to analyze the stability of the closed-loop dynamical system with an MTL framework to model the desired switching signals $\sigma_i\left(t\right)$. More specifically, we construct a switched system that cyclically switches between two modes, a stabilizable mode corresponding to targets being chased by the herder, and an uncontrolled unstable mode corresponding to targets not being chased that can potentially diverge from their goals. Using Lyapunov-based methods, the developed dwell-time conditions must be satisfied by the switching signals to ensure stability of the overall switched system. Then, these dwell-time conditions are encoded into MTL specifications and a switched nonlinear MPC problem is formulated to obtain the optimal switching signal $\sigma_i\left(t\right)$ that satisfies these MTL specifications.

To quantify the herding objective, a tracking error $e_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ for all $i \in T$, is defined by

$$e_i\left(t\right) \triangleq x_i\left(t\right) - x_{i,g}, \tag{3}$$

where, $x_{i,g} \in \mathbb{R}^n$ for all $i \in T$ denotes the desired goal location of the target agents. Due to the fact that the target agent dynamics in (1) do not explicitly contain a control input, a back-stepping strategy is used to inject a virtual control input into the dynamics of the target agents. The back-stepping error, denoted by $\eta : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, is defined by

$$\eta\left(t\right) \triangleq x_d\left(\sigma_i\left(t\right),t\right) - x_N\left(t\right), \tag{4}$$

where the herder agent's desired trajectory is denoted by $x_d : \{0,1\}^{(N-1)} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n$. The herder agent's desired trajectory is subsequently designed in the analysis of the proposed back-stepping strategy and enables the herder agent to control the group of target agents.

Based on the subsequent stability analysis of the back-stepping strategy, the herder agent tracks a desired trajectory. In doing so, the behavior of the herder agent indirectly influences the dynamics of the target agents to ensure the herding objective is achieved.

## IV. Controller Design

This section introduces the design of the desired herder trajectory and switching controller. Motivated by the subsequent optimal control formulation in Section VI, a discrete-time binary variable is defined for each target agent by $b_i : \mathbb{N}_{\geq 0} \to \{0,1\}$. At the discrete-time index $t_j \in \mathbb{N}_{\geq 0}$, the binary variable $b_i\left(t_j\right) = 1$ when the $i^{th}$ target agent is in the chased mode, and $b_i\left(t_j\right) = 0$ otherwise. The time period between discrete time instants is denoted by $T_s \in \mathbb{R}_{>0}$, i.e., at index $t_j$, the time instant is $t_j T_s$. Then, the switching signal is defined by

$$\sigma_i\left(t\right) \triangleq b_i\left(t_j\right), \quad \forall t_j \in \mathbb{N}_{\geq 0}, t \in [t_j T_s, (t_j+1)T_s). \tag{5}$$

Furthermore, to facilitate the subsequent switching analysis, the set of operating modes is defined as $\mathcal{P} \triangleq \{c,u\}$, where $c$ and $u$ denote the chased and unchased modes, respectively. Henceforth, the times $t_{i,k}^c \in \mathbb{R}_{\geq 0}$ and $t_{i,k}^u \in \mathbb{R}_{\geq 0}$ denote the $k^{th}$ instants when the $i^{th}$ target agent switches to the chased mode and unchased mode, respectively, where $k \in \mathbb{N}_{\geq 0}$ for all $i \in T$. Based on the switching instants, the duration of the $k^{th}$ activation of the chased and unchased modes for the $i^{th}$ target agent are defined as $\Delta t_{i,k}^c \triangleq t_{i,k}^u - t_{i,k}^c$ and $\Delta t_{i,k}^u \triangleq t_{i,k+1}^c - t_{i,k}^u$, respectively.

The desired trajectory for the herder agent acts as a virtual input to the target agent, and is designed as[3]

$$x_d \triangleq \sum_{i=1}^{N-1} \sigma_i \left(\frac{k_1}{\alpha_i} e_i + x_{i,g}\right), \tag{6}$$

where the control gain $k_1 \in \mathbb{R}_{>0}$ is defined as $k_1 \triangleq k_{1,a} + k_{2,a}$ for the subsequent analysis. The control input of the herder agent is designed as

$$u \triangleq \sum_{i=1}^{N-1} \sigma_i \left[k_2 \eta + \alpha_i e_i + \left(k_3 \left\|x_i - x_N\right\| + k_4\right) \operatorname{sgn}\left(\eta\right)\right], \tag{7}$$

where $k_2, k_3, k_4 \in \mathbb{R}_{>0}$ denote user-defined control gains, and $\operatorname{sgn}\left(\cdot\right)$ represents the signum function.

Based on the design of the switching signal, the $i^{th}$ target agent that is currently chased corresponds to $\sigma_i = 1$. To facilitate the analysis, the current chased target agent is denoted by $x^c$, where $x^c = x_i$ when the current target agent corresponds to the $i^{th}$ target agent, and $x^c = x_j$ otherwise (i.e., $j \in T \backslash i$). Similarly, the tracking error, desired goal, drift dynamics, and repulsive function associated with the current chased target agent denoted by $e^c$, $x_g^c$, $f^c$, and $\alpha^c$, respectively. It then follows, that the desired trajectory in (6) and switching controller in (7) can be expressed as $x_d \equiv \frac{k_1}{\alpha^c} e^c + x_g^c$ and $u \equiv k_2 \eta + \alpha^c e^c + \left(k_3 \left\|x^c - x_N\right\| + k_4\right) \operatorname{sgn}\left(\eta\right)$, respectively.

By taking the time derivative of the tracking error in (3), using the definition of the back-stepping error in (4), and substituting the target agent dynamics and desired herder state given by (1) and (6), respectively, the closed-loop tracking error system of the $i^{th}$ target agent is

$$\dot{e}_i = \begin{cases} \left(e_i + \eta - \frac{k_1}{\alpha_i} e_i\right) \alpha_i + f_i, & t \in [t_{i,k}^c, t_{i,k}^u), \\ \left(x_i + \eta - \frac{k_1}{\alpha_j} e_j - x_{j,g}\right) \alpha_i + f_i, & t \in [t_{i,k}^u, t_{i,k+1}^c). \end{cases} \tag{8}$$

[3]For notational brevity, functional dependence on states and time will be henceforth suppressed, except for when introducing new terms and where necessary for clarity.

By taking the time derivative of the back-stepping error (4) and substituting the control input in (7), yields the closed-loop back-stepping error system

$$\dot{\eta}=\begin{cases}k_1\left(x_i-x_N\right)+\frac{k_1}{\alpha_i}f_i-k_2\eta-\alpha_i e_i\\ -\left(k_3\left\|x_i-x_N\right\|+k_4\right)\operatorname{sgn}\left(\eta\right), & t\in[t^c_{i,k},t^u_{i,k}),\\ k_1\left(x_j-x_N\right)+\frac{k_1}{\alpha_j}f_j-k_2\eta-\alpha_j e_j\\ -\left(k_3\left\|x_j-x_N\right\|+k_4\right)\operatorname{sgn}\left(\eta\right), & t\in[t^u_{i,k},t^c_{i,k+1}).\end{cases} \tag{9}$$

## V. Switched System Stability Analysis

This section proposes a switched system analysis to develop dwell-time conditions that ensure stability of the closed-loop switched system in (8)-(9). The analysis first examines the behavior of the $i^{th}$ target agent's state trajectories when operating in each of the two modes. The analysis shows the $i^{th}$ target agent's state trajectories exponentially converge to a uniformly ultimate bound when in the chased mode. Then while in the unchased mode, its state trajectories are divergent. Based on the switching strategy, the target agents alternate between chased and unchased modes. A single cycle is considered as the time spanning both the chased and unchased modes, i.e., $t\in[t^c_{i,k},t^c_{i,k+1})$. By analyzing the recursive relation of a candidate Lyapunov function extending over $m\in\mathbb{N}_{\geq 0}$ cycles, sufficient dwell-time conditions are developed. These conditions ensure boundedness and convergence of the state trajectories, and require the $i^{th}$ target agent to remain in the chased mode by a time ratio of the activation time in the unchased mode.

The candidate Lyapunov function, denoted by $V_i:\mathbb{R}^{2n}\to\mathbb{R}_{\geq 0}$ for all $i\in T$, is defined by

$$V_i\left(z_i\right)\triangleq\frac{1}{2}e_i^Te_i+\frac{1}{2}\eta^T\eta=\frac{1}{2}\|z_i\|^2, \tag{10}$$

where $z_i\triangleq\left[e_i^T,\,\eta^T\right]^T$. The following lemma shows stability of the state trajectories when the $i^{th}$ target agent is operating in the chased mode.

**Lemma 1.** *During time $t\in[t^c_{i,k},t^u_{i,k})$ for all $k\in\mathbb{N}_{\geq 0}$ and $i\in T$, the virtual input in (6) and the herder control input in (7) ensures the $i^{th}$ target agent's state trajectories are bounded by*

$$\|z_i\|^2\leq C_{i,1}e^{-\lambda_{i,1}\left(t-t^c_{i,k}\right)}+\varepsilon_{i,1}, \tag{11}$$

*provided the following gain conditions are satisfied*

$$k_{1,a}>\overline{\alpha}_i,\,k_{1,b}>0,\,k_2>0,\,k_3>k_1,\,k_4>\frac{k_1\overline{f}_i}{\underline{\alpha}_i}, \tag{12}$$

*where $C_{i,1},\,\lambda_{i,1},\,\varepsilon_{i,1}\in\mathbb{R}_{>0}$ are known constants, and the constants $\overline{f}_i$, $\underline{\alpha_i}$, and $\overline{\alpha_i}$ are introduced in Assumptions 1 and 2, respectively.*

*Proof:* Taking the time derivative of the candidate Lyapunov function in (10), substituting the closed-loop error systems from (8) and (9), using Assumptions 1 and 2, and completing the squares yields

$$\dot{V}_i\leq-\lambda_{i,1}V_i+\delta_{i,1}, \tag{13}$$

where $\lambda_{i,1}\triangleq\;2\min\left\{\left(k_{1,a}-\overline{\alpha}_i\right),k_2\right\}$ and $\delta_{i,1}\triangleq\frac{\overline{f}_i^2}{4k_{1,b}}$ are known for all $i\in T$, provided the gain conditions in (12) are satisfied. The Comparison Lemma [28] is then used to solve (13). Then further upper-bounding yields

$$V_i\leq\mathrm{e}^{-\lambda_{i,1}\left(t-t^c_{i,k}\right)}V_i\left(z_i\left(t^c_{i,k}\right)\right)+\frac{\delta_{i,1}}{\lambda_{i,1}}. \tag{14}$$

From (10), the solution in (14) can be expressed as (11), where the positive constants $C_{i,1}\triangleq\left\|z_i\left(t^c_{i,k}\right)\right\|^2$ and $\varepsilon_{i,1}\triangleq\frac{2\delta_{i,1}}{\lambda_{i,1}}$. ■

The following lemma develops a maximum dwell-time condition. The condition ensures the $i^{th}$ target agent's state trajectories are bounded by a user-defined parameter while operating in the unchased mode.

**Lemma 2.** *During time $t\in[t^u_{i,k},t^c_{i,k+1})$ for all $k\in\mathbb{N}_{\geq 0}$ and $i\in T$, the virtual input in (6) and the herder control input in (7) ensures the $i^{th}$ target agent's state trajectories are bounded by*

$$\|z_i\|\leq\sqrt{2\overline{V}}, \tag{15}$$

*provided the following maximum dwell-time condition*

$$\Delta t^u_{i,k}\leq\frac{1}{\lambda_{i,2}}ln\left(\frac{\overline{V}+\frac{\delta_{i,2}}{\lambda_{i,2}}}{V\left(z\left(t^u_{i,k}\right)\right)}+\frac{\delta_{i,2}}{\lambda_{i,2}}\right), \tag{16}$$

*and the gain conditions in (12) are satisfied, where $\delta_{i,2},\lambda_{i,2}\in\mathbb{R}_{>0}$ are known constants, and the user-defined threshold is selected as $\overline{V}>V\left(z\left(t^u_{i,k}\right)\right)$.*

*Proof:* Taking the time derivative of the candidate Lyapunov function in (10), substituting the closed-loop error systems from (8) and (9), using Assumptions 1 and 2, and using Young's Inequality yields

$$\dot{V}_i\leq\lambda_{i,2}V_i+\delta_{i,2}, \tag{17}$$

where $\lambda_{i,2}\triangleq2\left(\frac{5\overline{\alpha}_i}{2}+\frac{\overline{\alpha}_ik_1}{2\underline{\alpha}_j}+\frac{1}{2}\right)$ and $\delta_{i,2}\triangleq\frac{\overline{\alpha}_i}{2}\left\|x_{i,g}\right\|^2+\frac{\overline{\alpha}_i}{2}\left\|x_{j,g}\right\|^2+\frac{1}{2}\overline{f}_i^2+\left(\frac{\overline{\alpha}_ik_1}{2\underline{\alpha}_j}+\frac{\overline{\alpha}_j}{2}\right)\left\|e_j\left(t^u_{i,k}\right)\right\|^2$ are known constants. The Comparison Lemma [28] is applied to (17) to yield

$$V_i\leq\mathrm{e}^{\lambda_{i,2}\left(t-t^u_{i,k}\right)}\left(V\left(z_i\left(t^u_{i,k}\right)\right)+\frac{\delta_{i,2}}{\lambda_{i,2}}\right). \tag{18}$$

To ensure the bound in (15) is satisfied, a sufficient condition is developed for the dwell-time $\Delta t^u_{i,k}$ for all $i\in T$ and $k\in\mathbb{N}_{\geq 0}$. Specifically, at the switching instant $t^c_{i,k+1}$, the solution (18) is bounded as

$$V_i\left(z\left(t^c_{i,k+1}\right)\right)\leq\mathrm{e}^{\lambda_{i,2}\Delta t^u_{i,k}}\left(V\left(z_i\left(t^u_{i,k}\right)\right)+\frac{\delta_{i,2}}{\lambda_{i,2}}\right)\leq\overline{V}. \tag{19}$$

Then, solving the inequality in (19) results in the maximum dwell-time condition in (16), provided the user-defined parameter is selected as $\overline{V}>V\left(z\left(t^u_{i,k}\right)\right)$ for all $i\in T$ and $k\in N_{\geq 0}$. Provided the maximum dwell-time condition in (16) is satisfied, using the definition of the Lyapunov function in (10), the solution (19) is expressed as the bound in (15). ■

In Lemmas 1 and 2, a single cycle of the chased and unchased modes were analyzed, respectively. It was shown that the state trajectories of the $i^{th}$ target agent are exponentially convergent and divergent when operating in the chased and unchased modes, respectively. Based on the convergence and divergence rates, Theorem 1 develops sufficient dwell-time conditions to ensure the herding objective is achieved despite the unstable mode. The developed dwell-time conditions will ensure that over consecutive cycles of switching instants, i.e., $t \in [t^c_{i,km}, t^c_{i,(k+1)m})$ where $m \in \mathbb{N}_{\geq 0}$, the state trajectories will converge to a stationary point. Additionally, because the state trajectories can grow between consecutive cycles of switching instants, the dwell-time conditions will ensure the state trajectories remain bounded by a user-defined parameter.

To facilitate the analysis, the total activation time of the chased and unchased modes between switching instants for the $i^{th}$ target agent are denoted by $T^c_i(a,b) = \sum_{l=a}^{b} \Delta t^c_{i,l}$ and $T^u_i(a,b) = \sum_{l=a}^{b} \Delta t^u_{i,l}$, respectively, where $T^c_i, T^u_i : \mathbb{N}_{\geq 0} \times \mathbb{N}_{\geq 0} \to \mathbb{R}_{\geq 0}$ and $a, b \in \mathbb{N}_{\geq 0}$.

**Theorem 1.** *The virtual input in (6) and the herder control input in (7) ensure the $i^{th}$ target agent's state trajectories are bounded for all time $t \in \mathbb{R}_{\geq 0}$ such that*

$$\limsup_{t\to\infty} \|z_i(t)\|^2 \leq V^*_i e^{\lambda_{2,i}\Delta t^u_{i,k}}, \tag{20}$$

*provided there exists an $m < \infty$, sequences $\left\{\Delta t^c_{i,k}\right\}_{k=0}^{\infty}$ and $\left\{\Delta t^u_{i,k}\right\}_{k=0}^{\infty}$ such that the maximum dwell-time condition in (16) is satisfied for all $k \in \mathbb{N}_{\geq 0}$, and the sufficient condition*

$$T^c(km, (k+1)m-1) > \frac{\lambda_{i,2}}{\lambda_{i,1}} T^u(km, (k+1)m-1) \tag{21}$$

*is satisfied.*

*Proof:* Consider a single cycle over the duration $t \in [t^c_{i,km}, t^c_{i,km+1})$. From Lemmas 1 and 2, the solutions of the Lyapunov equation in the chased and unchased modes given by (14) and (18), respectively, are used to yield

$$V_i\left(z_i\left(t^c_{i,km+1}\right)\right) \leq \mathrm{e}^{\lambda_{i,2}\Delta t^u_{i,km} - \lambda_{i,1}\Delta t^c_{i,km}} V_i\left(z_i\left(t^c_{i,km}\right)\right) + \mathrm{e}^{\lambda_{i,2}\Delta t^u_{i,km}}\left(\frac{\delta_{i,1}}{\lambda_{i,1}} + \frac{\delta_{i,2}}{\lambda_{i,2}}\right), \tag{22}$$

which provides a difference equation for the value of the Lyapunov function spanning a single cycle. Then using (14), (18), and (22), a recursive relation over $m$ cycles yields

$$V_i\left(z_i\left(t^c_{i,(k+1)m}\right)\right) \leq C_{i,3} V_i\left(z_i\left(t^c_{i,km}\right)\right) + \varepsilon_{i,3}, \tag{23}$$

where the positive constants $C_{i,3}, \varepsilon_{i,3} \in \mathbb{R}_{\geq 0}$ and $C_{i,3} \triangleq \mathrm{e}^{\lambda_{i,2}T^u(km,(k+1)m-1) - \lambda_{i,1}T^c(km,(k+1)m-1)}$ are known. To ensure the expression in (23) is a contraction, the inequality $|C_{i,3}| < 1$ is enforced and solved to yield the sufficient condition in (21). Provided the difference equation (23) is a contraction, then $V_i\left(z_i\left(t^c_{i,(k+1)m}\right)\right)$ converges to the fixed point $V^*_i \triangleq \frac{\varepsilon_{i,3}}{1-C_{i,3}}$. Additionally, because the dwell-time condition in (21) is specified over $m$ cycles, $V_i$ may grow within $t \in [t^c_{i,km}, t^c_{i,(k+1)m})$. If the maximum dwell-time condition (16) is satisfied for all $k \in \mathbb{N}_{\geq 0}$, then using (10), the state trajectories are bounded by (20). ■

## VI. Switching Protocol Synthesis

In this section, a switching protocol is synthesized for the herder's control input in (7). MTL formulas encode the sufficient dwell-time conditions developed in Section V given by (16) and (21). Then the encoded MTL formulas is used as constraints to a nonlinear optimization formulation. The solution to the nonlinear optimization formulation yields a switching signal that minimizes a cost function while satisfying sufficient dwell-time conditions in (16) and (21).

The maximum dwell-time condition in (16) is encoded by the following MTL formula

$$\phi_1 \triangleq \bigwedge_{i\in T} \Box\Diamond_{[0,r_i]} b_i(t_j), \tag{24}$$

where $r_i \in \mathbb{N}_{\geq 0}$ is a positive integer such that the upper-bound $\frac{1}{\lambda_{i,2}}\ln\left(\frac{\overline{V}+\frac{\delta_{i,2}}{\lambda_{i,2}}}{V\left(z\left(t^u_{i,k}\right)\right)+\frac{\delta_{i,2}}{\lambda_{i,2}}}\right)$ in (16) is in the interval $[r_i T_s, (r_i+1)T_s)$. The MTL formula $\phi_1$ specifies that every target agent should be chased at least once every $r_i$ time steps.

The dwell-time ratio condition in (21) is specified over $m$ cycles of switching through both the chased and unchased modes. To encode the dwell-time condition as an MTL formula, a sufficient condition over one single cycle ($m = 1$) is required. A sufficient dwell-time condition over a cycle yields[4]

$$\Delta t^c_{i,k} > \frac{\lambda_{i,2}}{\lambda_{i,1}} \Delta t^u_{i,k}. \tag{25}$$

Since the duration $\Delta t^c_{i,k}$ for the chased mode is a ratio of the maximum duration $\Delta t^u_{i,k}$, substituting (16) into (25) yields

$$\Delta t^c_{i,k} \geq \frac{1}{\lambda_{i,1}}\ln\left(\frac{\overline{V}+\frac{\delta_{i,2}}{\lambda_{i,2}}}{V\left(z\left(t^u_{i,k}\right)\right)+\frac{\delta_{i,2}}{\lambda_{i,2}}}\right), \tag{26}$$

which offers a lower-bound for the duration $\Delta t^c_{i,k}$. The MTL formula that capture the sufficient condition are formulated as

$$\phi_2 \triangleq \bigwedge_{i\in T} \Box(\neg b_i(t_j - 1) \wedge b_i(t_j) \to \Box_{[1,s_i]} b_i(t_j)), \tag{27}$$

where $s_i \in \mathbb{N}_{\geq 0}$ is a positive integer such that the lower bound $\frac{1}{\lambda_{i,1}}\ln\left(\frac{\overline{V}+\frac{\delta_{i,2}}{\lambda_{i,2}}}{V\left(z\left(t^u_{i,k}\right)\right)+\frac{\delta_{i,2}}{\lambda_{i,2}}}\right)$ is in the interval $[(s_i-1)T_s, s_i T_s)$. The MTL formula $\phi_2$ specifies that once a target agent enters the chased mode, it should remain in that mode for at least $s_i$ time steps. Then, a mixed-integer nonlinear program over a receding-horizon at any time index $l \in \mathbb{N}_{\geq 0}$ is formulated as

[4]By construction, if every individual cycle of switching (i.e., $m = 1$) satisfies the dwell-time ratio condition in (21), then it follows for any $m < \infty$ cycles (21) is satisfied.

$$\underset{\mathbf{b}}{\operatorname{argmin}} \quad J(\mathbf{b}) = \sum_{i=1}^{N-1} \|x_i(l+n-1) - x_{i,g}\|^2 \tag{28}$$

subject to:

$$\dot{x}_N(t) = \sum_{i=1}^{N-1} \sigma_i [k_2 \eta + \alpha_i e_i + (k_3 \|x_i - x_N\| + k_4) \operatorname{sgn}(\eta)], \tag{29}$$

$$\dot{x}_i(t) = (x_i - x_N)\alpha_i + f_i, \quad \forall i \in T, \tag{30}$$

$$\sum_{i=1}^{N-1} b_i(t_j) = 1, \quad \forall t_j \in \{l, \ldots, l+n-1\}, \tag{31}$$

$$[\mathbf{b}^l, \mathbf{b}], 0 \models_W \phi_1 \wedge \phi_2, \tag{32}$$

where $\mathbf{b} \triangleq [b_1(l : l+n-1), b_2(l : l+n-1), \ldots, b_{n-1}(l : l+n-1)]$ and $\mathbf{b}^l$ is the sequence of switching signals that have been determined up to the current time index $l$. In (16), due to the term $V(t_{i,k}^u)$, the dwell-time condition changes each time the system transitions from the chased to unchased modes. Therefore, after solving the nonlinear program at time index $t_j$, the corresponding $r_i$ and $s_i$ in MTL specifications $\phi_1$ and $\phi_2$ are updated depending on whether $t_j$ corresponds to a switching time instant.

**Theorem 2.** *With the desired herder trajectory in* (6)*, the herder control input in* (7)*, and the synthesized switching signal in* (5) *from solving the nonlinear optimal control problem given in (28)-(32), the state trajectories are bounded by* (20) *for all time* $t \in \mathbb{R}_{\geq 0}$.

*Proof:* If there exists a feasible solution to the nonlinear program in (28)-(32) at any time index $t_j \in \mathbb{N}_{\geq 0}$, then the resulting switching signal in (5) will weakly satisfy the MTL specifications in (24) and (27) simultaneously. Thus the synthesized switching signal satisfies the dwell-time conditions in (16) and (21). Then by Theorem 1, the state trajectories are bounded by (20) for all time $t \in \mathbb{R}_{\geq 0}$. ∎

## VII. Conclusion

Single-agent indirect herding of multiple agents for a class of nonlinear switched systems is achieved. A Lyapunov-based switching analysis is leveraged to develop dwell-time criteria for stability. Then using an MTL approach, a switching protocol is synthesized by solving a mixed-integer nonlinear optimal control formulation. The developed switching protocol minimizes a cost function while constrained to the nonlinear dynamics and MTL specifications. Future efforts include exploiting properties of persistent feasibility solutions of the nonlinear MPC, extending results to learning-based controllers where model uncertainty is present, and considering a network of multiple cooperative herder agents.

## References


[1] M. Mesbahi and M. Egerstedt, *Graph theoretic methods in multiagent networks*. Princeton University Press, 2010.

[2] J.-M. Lien, S. Rodriguez, J.-P. Malric, and N. M. Amato, "Shepherding behaviors with multiple shepherds," in *Proc. IEEE Int. Conf. on Robot. and Autom.*, 2005, pp. 3402–3407.

[3] R. Vaughan, N. Sumpter, J. Henderson, A. Frost, and S. Cameron, "Experiments in automatic flock control," *Robot. Autom. Syst.*, vol. 31, no. 1, pp. 109–117, 2000.

[4] T. Miki and T. Nakamura, "An effective simple shepherding algorithm suitable for implementation to a multi-mobile robot system," in *Proc. of the First Int. Conf. on Innov. Computing, Inf., and Control*, vol. 3, Washington, DC, USA, 2006, pp. 161–165.

[5] D. Strömbom, R. P. Mann, A. M. Wilson, S. Hailes, A. J. Morton, D. J. T. Sumpter, and A. J. King, "Solving the shepherding problem: heuristics for herding autonomous, interacting agents," *J. of the R. Society Interface*, vol. 11, no. 100, 2014.

[6] A. Pierson and M. Schwager, "Controlling noncooperative herds with robotic herders," *IEEE Trans. Robot.*, vol. 34, no. 2, pp. 517–525, 2018.

[7] V. Chipade and D. Panagou, "Herding an adversarial attacker to a safe area for defending safety-critical infrastructure," in *Proc. Am. Control Conf.*, 2019, pp. 1035–1041.

[8] R. Licitra, Z. Bell, and W. Dixon, "Single agent indirect herding of multiple targets with unknown dynamics," *IEEE Trans. Robotics*, vol. 35, no. 4, pp. 847–860, 2019.

[9] R. Licitra, Z. Hutcheson, E. Doucette, and W. E. Dixon, "Single agent herding of n-agents: A switched systems approach," in *IFAC World Congr.*, 2017, pp. 14 374–14 379.

[10] R. Licitra, Z. I. Bell, E. Doucette, and W. E. Dixon, "Single agent indirect herding of multiple targets: A switched adaptive control approach," *IEEE Control Syst. Lett.*, vol. 2, no. 1, pp. 127–132, January 2018.

[11] R. G. Goebel, R. Sanfelice, and A. R. Teel, *Hybrid Dynamical Systems*. Princeton University Press, 2012.

[12] D. Liberzon, *Switching in Systems and Control*. Birkhauser, 2003.

[13] S. Luo, F. Deng, and W. Chen, "Unified dwell time-based stability and stabilization criteria for switched linear stochastic systems and their application to intermittent control," *Int. J. of Robust Nonlinear Control*, vol. 28, pp. 2530–2537, 2012.

[14] P. Mhaskar, N. H. El-Farra, and P. D. Christofides, "Predictive control of switched nonlinear systems with scheduled mode transitions," *IEEE Trans. Autom. Control*, vol. 50, pp. 1670–1680, 2005.

[15] P. Polterauer, G. P. Incremona, P. Colancri, and L. del Re, "A switching nonlinear mpc approach for ecodriving," in *Proc. Am. Control Conf.* IEEE, 2019, pp. 4608–4613.

[16] R. Koymans, "Specifying real-time properties with metric temporal logic," *Real-time systems*, vol. 2, no. 4, pp. 255–299, 1990.

[17] Z. Xu, C. Belta, and A. Julius, "Temporal logic inference with prior information: An application to robot arm movements," *IFAC-PapersOnLine*, vol. 48, no. 27, pp. 141–146, 2015.

[18] S. Saha and A. A. Julius, "Task and motion planning for manipulator arms with metric temporal logic specifications," *IEEE robotics and automation letters*, vol. 3, no. 1, pp. 379–386, 2017.

[19] Y. V. Pant, H. Abbas, and R. Mangharam, "Smooth operator: Control using the smooth robustness of temporal logic," in *2017 IEEE Conference on Control Technology and Applications (CCTA)*. IEEE, 2017, pp. 1235–1240.

[20] S. Saha and A. A. Julius, "An milp approach for real-time optimal controller synthesis with metric temporal logic specifications," in *2016 American Control Conference (ACC)*. IEEE, 2016, pp. 1105–1110.

[21] J. Fu and U. Topcu, "Computational methods for stochastic control with metric interval temporal logic specifications," in *2015 54th IEEE Conference on Decision and Control (CDC)*. IEEE, 2015, pp. 7440–7447.

[22] S. Andersson, A. Nikou, and D. V. Dimarogonas, "Control synthesis for multi-agent systems under metric interval temporal logic specifications," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 2397–2402, 2017.

[23] S. Karaman and E. Frazzoli, "Vehicle routing problem with metric temporal logic specifications," in *2008 47th IEEE Conference on Decision and Control*. IEEE, 2008, pp. 3953–3958.

[24] R. Alur and D. L. Dill, "A theory of timed automata," *Theoretical computer science*, vol. 126, no. 2, pp. 183–235, 1994.

[25] R. M. M. Eric M. Wolff, Ufuk Topcu, "Optimization-based trajectory generation with linear temporal logic specifications," in *Proc. Int. Conf. Rob. Aut.*, 2014, pp. 5319–5325.

[26] Z. Xu, F. M. Zegers, B. Wu, W. E. Dixon, and U. Topcu, "Controller synthesis for multi-agent systems with intermittent communication: A metric temporal logic approach," in *Proc. Allerton Conf. on Commun., Control, and Compu.*, Sept. 2019.

[27] O. Maler and D. Nickovic, "Monitoring temporal properties of continuous signals," in *Formal Techniques, Modelling and Analysis of Timed and Fault-Tolerant Systems*. Springer, 2004, pp. 152–166.

[28] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Upper Saddle River, NJ: Prentice Hall, 2002.