Connectivity of Dynamic Graphs

Michael M. Zavlanos and George J. Pappas

December 2, 2013

1 Abstract

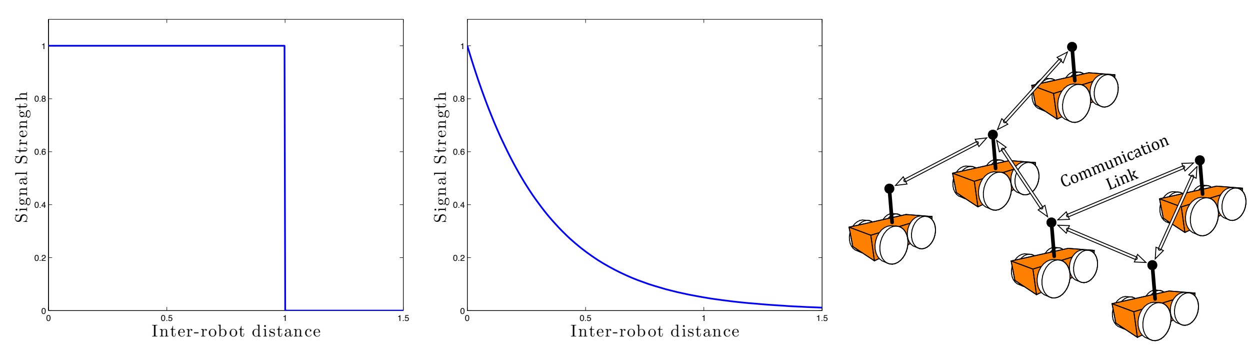
Dynamic networks have recently emerged as an efficient way to model various forms of interaction within teams of mobile agents, such as sensing and communication. This article focuses on the use of graphs as models of wireless communications. In this context, graphs have been used widely in the study of robotic and sensor networks and have provided an invaluable modeling framework to address a number of coordinated tasks ranging from exploration, surveillance and reconnaissance, to cooperative construction and manipulation. In fact, the success of these stories has almost always relied on efficient information exchange and coordination between the members of the team, as seen, e.g., in the case of distributed state agreement where multi-hop communication has been proven necessary for convergence and performance guarantees.

2 Keywords and Phrases

Algebraic graph theory,graph connectivity, distributed and hybrid control, convex optimization.

3 Introduction

Communication in networked dynamical systems has typically relied on constructs from graph theory, with disc-based and weighted-proximity graphs gaining the most popularity; see Figs. 1(a) and 1(b). Besides their simplicity, these models owe their popularity to their resemblance to radio



(a) (b) (c)

Figure 1: Fig. 1(a): Disc-based model of communication; Fig. 1(b): Weighted, proximity-based model of communication; Fig. 1(c): Connected network of mobile robots.

signal strength models, where the signals attenuate with the distance [1–3]. In this context, multi-hop communication becomes equivalent to network connectivity, defined as the property of a graph to transmit information between any pair of its nodes; see Fig. 1(c).

Specifically, let $\mathcal{G}(t) = \{\mathcal{V}, \mathcal{E}(t), \mathcal{W}(t)\}$ denote a graph on n nodes that can be robots or mobile sensors, so that $\mathcal{V} = \{1, \dots, n\}$ is the set of vertices, $\mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges at time t, and $\mathcal{W}(t) = \{w_{ij}(t) \mid (i,j) \in \mathcal{V} \times \mathcal{V}\}$ is a set of weights so that $w_{ij}(t) = 0$ if $(i,j) \notin \mathcal{E}(t)$ and $w_{ij}(t) > 0$ otherwise. If $w_{ij}(t) = w_{ji}(t)$ for all pairs of nodes i, j, then the graph is called undirected; otherwise it is called directed. The weights in $\mathcal{W}(t)$ typically model signal strength or channel reliability, as per the disc-based and weighted-proximity models in Figs. 1(a) and 1(b). In these models communication between nodes is related to their pairwise distance, giving rise to the dynamic or time-varying nature of the graph $\mathcal{G}(t)$ due to node mobility. Given an undirected dynamic graph $\mathcal{G}(t)$, we say that this graph is *connected* at time t if there exists a path, i.e., a sequence of distinct vertices such that consecutive vertices are adjacent, between any two vertices in $\mathcal{G}(t)$. In the case of directed graphs, two notions of connectivity are defined. A directed graph $\mathcal{G}(t)$ is called *strongly connected* if there exists a directed path between any two of its vertices; equivalently, if every vertex is reachable from any other vertex. On the other hand, a directed graph is called *weakly connected* if replacing all directed edges by undirected edges produces a connected undirected graph. Finally, a collection of graphs $\{\mathcal{G}(t) \mid t = t_0, \dots, t_k\}$ is called *jointly connected* over time if the union graph $\cup_{t=t_0}^{t_k} \mathcal{G}(t) = \{\mathcal{V}, \cup_{t=t_0}^{t_k} \mathcal{E}(t)\}$ is connected. Obviously, checking for the existence of paths between all pairs of nodes in a graph is difficult, especially so as the number of nodes in the graph increases. For this reason, equivalent, algebraic representations of graphs are employed that allow for efficient algebraic ways to check for connectivity, as we discuss in the following section.

While connectivity is necessary for information propagation in a network, it is also relevant to the performance of many networked dynamical processes, such as synchronization and gossiping, via its relation to the network eigenvalue spectra [4]. For example, the spectrum of the Laplacian matrix of a network plays a key role in the analysis of synchronization in networks of nonlinear oscillators [5, 6], distributed algorithms [7], and decentralized control problems [8, 9]. Similarly, the spectrum of the adjacency matrix determines the speed of viral information spreading in a network [10]. Additionally, more robust versions of connectivity, such as *k-node* or *k-edge* connectivity can be used to introduce robustness of a network to node or link failures, respectively [11, 12].

4 Graph-Theoretic Connectivity Control

4.1 Connectivity Using the Graph Laplacian Matrix

A metric that is typically employed to capture connectivity of dynamic networks is the second smallest eigenvalue $\lambda_2(L)$ of the Laplacian matrix $L \in \mathbb{R}^{n \times n}$ of the graph, also known as the algebraic connectivity or Fiedler value of the graph. For a weighted graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$ the entries of the Laplacian matrix are typically related to the weights in $\mathcal{W}$ so that the i, j entry of L is given by $[L]_{ij} = \sum_{j=1}^{n} w_{ij}$ if $i = j$ and $[L]_{ij} = -w_{ij}$ if $i \neq j$. The Laplacian matrix of an undirected graph is always a symmetric, positive semidefinite matrix whose smallest eigenvalue $\lambda_1(L)$ is identically zero with corresponding eigenvector the vector of all entries equal to one. Additionally, the algebraic connectivity $\lambda_2(L)$ is a concave function of the Laplacian matrix that is positive if and only if the graph is connected [13–16].

As the algebraic connectivity $\lambda_2(L)$ plays a critical role in determining whether a graph is connected or not, a number of methods have been proposed for its decentralized estimation and control. These range from methods that employ market-based control to underestimate the algebraic connectivity and accordingly control the network structure [12], to methods that enforce the states of the nodes to oscillate at frequencies that correspond to the Laplacian eigenvalues and then use Fast Fourier Transform to estimate these eigenvalues [17], to methods that iteratively update the interval where the algebraic connectivity is supposed to lie [18], and methods that rely on the Power Iteration method and its variants [19–24]. All the above techniques are often integrated with appropriate controllers to regulate mobility of the nodes while ensuring connectivity of the network. Another way that $\lambda_2(L)$ can be used to ensure connectivity of dynamic graphs is via optimization-based methods that maximize it away from its zero value. Such approaches were initially centralized as connectivity is a global property of a graph [25], although recently distributed subgradient algorithms [19] as well as non-iterative decomposition techniques [26] have also been proposed. As the algebraic connectivity is a non-differentiable function of the Laplacian matrix, designing continuous-time feedback controllers to maintain it positive definite is a challenging task. This problem was overcome in [27] via the use of gradient flows that maintain positive definiteness of the determinant of the projected Laplacian matrix to the space that is perpendicular to eigenvector of ones.

4.2 Connectivity Using the Graph Adjacency Matrix

Alternatively, connectivity can be captured by the sum of powers $\sum_{k=0}^{K} A^k$ of the adjacency matrix $A \in \mathbb{R}^{n\times n}$ of the network for $K \leq n-1$. The entries of the adjacency matrix are typically related to the weights in $\mathcal{W}$ as $[A]_{ij} = w_{ij}$. For disc-based graphs as in Fig. 1(a), the i, j entry of the kth power of the adjacency matrix $[A^k]_{ij}$ captures the number of paths of length k between nodes i and j; for weighted graphs, $[A^k]_{ij}$ captures a weighted sum of those paths. Therefore, the entries of $\sum_{k=0}^{K} A^k$ represent the number of paths up to length K between every pair of nodes in the graph [16]. By definition of graph connectivity, if all entries of $\sum_{k=0}^{K} A^k$ are positive for $K = n-1$, then the network is connected. Clearly, for $K < n-1$, not all entries of $\sum_{k=0}^{K} A^k$ are necessarily positive, even if the graph is connected. Maintaining positive definiteness of the positive entries of $\sum_{k=0}^{K} A^k$ of an initially connected graph maintains paths of length K between the corresponding nodes and, as shown in [11], is sufficient to maintain connectivity of the graph throughout.

The ability to capture graph connectivity using the adjacency matrix has given rise to optimization-based connectivity controllers [11, 28], that are often centralized due to the multi-hop dependencies between nodes due to the powers of the adjacency matrix. Since smaller powers correspond to shorter dependencies (paths), decentralization is possible as K decreases. If $K = 1$, connectivity maintenance reduces to preserving the pairwise links between the nodes in an initially connected network. Since the adjacency matrix of weighted graphs is often a differentiable function, this approach can result in continuous-time feedback solution techniques. Discrete-time approaches are discussed in [29–31], while [32–37] rely on local gradients that may also incorporate switching in the case of link additions. Switching between arbitrary spanning topologies has also been studied with the spanning subgraphs being updated by local auctions [12], distributed spanning tree algorithms [38], combination of information dissemination algorithms and graph picking games [39], or intermediate rendezvous [40, 41]. This class of approaches are typically hybrid, combining continuous link maintenance and discrete topology control. The algebraic connectivity $\lambda_2(L)$ and number of paths $\sum_{k=0}^{K} A^k$ metrics can also be combined to give controllers that maintain connectivity, while enforcing desired multi-hop neighborhoods for all agents [42].

A recent, comprehensive survey on graph-theoretic approaches for connectivity control of dynamic graphs can be found in [43].

5 Applications in Mobile Robot Network Control

Methods to control connectivity of dynamic graphs have been successfully applied to multiple scenarios that require network connectivity to achieve a global coordinated objective. Indicative of the impact of this work is recent literature on connectivity preserving rendezvous [29, 33, 37, 44, 45], flocking [36, 46] and formation control [37, 40], where so far connectivity had been an assumption. Further extensions and contributions involve connectivity control for double integrator agents [30], agents with bounded inputs [47–49] and indoor navigation [42], as well as for communication based on radio signal strength [50–53] and visibility constraints [29, 44, 54–56]. Periodic connectivity for robot teams that need to occasionally split in order to achieve individual objectives [57, 58] and sufficient conditions for connectivity in leader-follower networks [59], also add to the list. Early experimental results have demonstrated efficiency of these algorithms also in practice [57, 60, 61].

6 Summary and Future Directions

Although graphs provide a simple abstraction of inter-robot communications, it has long been recognized that since links in a wireless network do not entail tangible connections, associating links with arcs on a graph can be somewhat arbitrary. Indeed, topological definitions of connectivity start by setting target signal strengths to draw the corresponding graph. Even small differences in target strengths might result in dramatic differences in network topology [62]. As a result, graph connectivity is necessary but not nearly sufficient to guarantee communication integrity, interpreted as the ability of a network to support desired communication rates.

To address these challenges, a new body of work is recently appearing that departs from traditional graph-based models of communication. Specifically, [63] employs a simple, yet effective, modification that relies on weighted graph models with weights that capture the packet error probability of each link [64]. When using reliabilities as link metrics it is possible to model routing and scheduling problems as optimization problems that accept link reliabilities as inputs [65, 66]. The key idea proposed in [63] is to define connectivity in terms of communication rates and to use optimization formulations to describe optimal operating points of wireless networks. Then, the communication variables are updated in discrete time via a distributed gradient descent algorithm on the dual function, while robot motion is regulated in continuous time by means of appropriate distributed barrier potentials that maintain desired communication rates. Related approaches consider optimal communications based on T-slot time averages of the primal variables for general mobility schemes was recently addressed in [67], as well as optimization of mobility and communications based on the end-to-end bit error rate between nodes [68, 69].

7 Cross-References

- link to "Graphs for modeling networked control systems," Mesbahi Mesbahi and Magnus Egerstedt
- link to "Flocking in Networked Systems," Ali Jadbababie

References

[1] A. Neskovic, N. Neskovic, and G. Paunovic, "Modern approaches in modeling of mobile radio systems propagation environment," *IEEE Communications Surveys*, vol. 3, no. 3, pp. 1–12, 2000.

[2] J. D. Parsons, *The Mobile Radio Propagation Channel*. John Willey & Sons, 2000.

[3] K. Pahlavan and A. H. Levesque, *Wireless Information Networks*. New York, NY: John Willey & Sons, 1995.

[4] V. Preciado, "Spectral analysis for stochastic models of large-scale complex dynamical networks," Ph.D. dissertation, Department of Electrical Engineering and Computer Science, MIT, 2008.

[5] L. Pecora and T. Carrollg, "Master stability functions for synchronized coupled systems," *Physics Review Letters*, vol. 80, pp. 2109–2112, 1998.

[6] V. Preciado and G. Verghese, "Synchronization in generalized erdös-rényi networks of nonlinear oscillators," in *44th IEEE Conference on Decision and Control*, 2005, pp. 4628–463.

[7] N. Lynch, *Distributed Algorithms*. Morgan Kaufmann Publishers, 1997.

[8] A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Transactions on Automatic Control*, vol. 49, pp. 1465–1476, 2004.

[9] R. Olfati Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, pp. 1520–1533, 2004.

[10] P. Van Mieghem, J. Omic, and R. Kooij, "Virus spread in networks," *IEEE/ACM Transactions on Networking*, vol. 17, no. 1, pp. 1–14, 2009.

[11] M. M. Zavlanos and G. J. Pappas, "Controlling connectivity of dynamic graphs," in *Proc. 44th IEEE Conference on Decision and Control and European Control Conference*, Seville, Spain, December 2005, pp. 6388–6393.

[12] ——, "Distributed connectivity control of mobile networks," *IEEE Transactions on Robotics*, vol. 24, no. 6, pp. 1416–1428, December 2008.

[13] M. Fiedler, "Algebraic connectivity of graphs," *Czechoslovak Mathematical Journal*, vol. 23, no. 98, pp. 298–305, 1973.

[14] B. Mohar, "The laplacian spectrum of graphs," in *Graph Theory, Combinatorics, and Applications*, Y. Alavi, G. Chartrand, O. Ollermann, and A. Schwenk, Eds. New York, NY: Wiley, 1991, pp. 871–898.

[15] R. Merris, "Laplacian matrices of a graph: A survey," *Linear Algebra its Applications*, vol. 197, pp. 143–176, 1994.

[16] C. Godsil and G. Royle, *Algebraic Graph Theory*, ser. Graduate Texts in Mathematics. Berlin, Germany: Springer-Verlag, 2001, vol. 207.

[17] M. Franceschelli, A. Gasparri, A. Giua, and C. Seatzu, "Decentralized estimation of laplacian eigenvalues in multi-agent systems," *Automatica*, vol. 49, no. 4, pp. 1031–1036, April 2013.

[18] E. Montijano, J. I. Montijano, and C. Sagues, "Adaptive consensus and algebraic connectivity estimation in sensor networks with chebyshev polynomials," in *In Proc. 50th IEEE Conference on Decision and Control*, Orlando, FL, December 2011, pp. 4296–4301.

[19] M. C. DeGennaro and A. Jadbabaie, "Decentralized control of connectivity for multi-agent systems," in *Proc. 45th IEEE Conference on Decision and Control*, San Diego, CA, December 2006, pp. 3628–3633.

[20] D. Kempe and F. McSherry, "A decentralized algorithm for spectral analysis," *Journal of Computer and System Sciences*, vol. 74, no. 1, pp. 70–83, 2008.

[21] B. N. Oreshkin, M. J. Coates, and M. G. Rabbat, "Optimization and analysis of distributed averaging with short node memory," *IEEE Transactions on Signal Processing*, vol. 58, no. 5, pp. 2850–2865, 2010.

[22] L. Sabattini, N. Chopra, and C. Secchi, "On decentralized connectivity maintenance for mobile robotic systems," in *In Proc. 50th IEEE Conference on Decision and Control*, Orlando, FL, December 2011, pp. 988–993.

[23] P. Yang, R. A. Freeman, G. J. Gordon, K. M. Lynch, S. S. Srinivasa, and R. Sukthankar, "Decentralized estimation and control of graph connectivity for mobile sensor networks," *Automatica*, vol. 46, no. 2, pp. 390–396, 2010.

[24] F. Knorn, R. Stanojevic, M. Corless, and R. Shorten, "A framework for decentralized feedback connectivity control with application to sensor networks," *International Journal of Control*, vol. 82, no. 11, pp. 2095–2114, 2009.

[25] Y. Kim and M. Mesbahi, "On maximizing the second smallest eigenvalue of a state-dependent graph laplacian," *IEEE Transactions on Automatic Control*, vol. 51, no. 1, pp. 116–120, January 2006.

[26] A. Simonetto, T. Kaviczky, and R. Babuska, "Constrained distributed algebraic connectivity maximization in robotic networks," *Automatica*, vol. 49, no. 5, pp. 1348–1357, May 2013.

[27] M. M. Zavlanos and G. J. Pappas, "Potential fields for maintaining connectivity of mobile networks," *IEEE Transactions on Robotics*, vol. 23, no. 4, pp. 812–816, August 2007.

[28] K. Srivastava and M. W. Spong, "Multi-agent coordination under connectivity constraints," in *Proc. 2008 American Control Conference*, Seattle, WA, June 2008, pp. 2648–2653.

[29] H. Ando, Y. Oasa, I. Suzuki, and M. Yamashita, "Distributed memoryless point convergence algorithm for mobile robots with limited visibility," *IEEE Transactions on Robotics and Automation*, vol. 15, no. 5, pp. 818–828, October 1999.

[30] G. Notarstefano, K. Savla, F. Bullo, and A. Jadbabaie, "Maintaining limited-range connectivity among second-order agents," in *Proc. 2006 American Control Conference*, Minneapolis, MN, June 2006, pp. 2124–2129.

[31] F. Bullo, J. Cortes, and S. Martinez, *Distributed Control of Robotic Networks*, ser. Applied Mathematics Series. Princeton University Press, 2009.

[32] D. P. Spanos and R. M. Murray, "Robust connectivity of networked vehicles," in *Proc. 43rd IEEE Conference on Decision and Control*, Bahamas, December 2004, pp. 2893–2898.

[33] D. V. Dimarogonas and K. J. Kyriakopoulos, "Connectedness preserving distributed swarm aggregation for multiple kinematic robots," *IEEE Transactions on Robotics*, vol. 24, no. 5, pp. 1213–1223, October 2008.

[34] A. Cornejo and N. Lynch, "Connectivity service for mobile ad-hoc networks," in *Proc. 2nd IEEE International Conference on Self-Adaptive and Self-Organizing Systems Workshops*, October 2008, pp. 292–297.

[35] Z. Yao and K. Gupta, "Backbone-based connectivity control for mobile networks," in *Proc. IEEE International Conference on Robotics and Automation*, Kobe, Japan, May 2009, pp. 1133–1139.

[36] M. M. Zavlanos, A. Jadbabaie, and G. J. Pappas, "Flocking while preserving network connectivity," in *Proc. 46th IEEE Conference on Decision and Control*, New Orleans, LA, December 2007, pp. 2919–2924.

[37] M. Ji and M. Egerstedt, "Coordination control of multi-agent systems while preserving connectedness," *IEEE Transactions on Robotics*, vol. 23, no. 4, pp. 693–703, August 2007.

[38] J. Wagenpfeil, A. Trachte, T. Hatanaka, M. Fujita, and O. Sawodny, "A distributed minimum restrictive connectivity maintenance algorithm," in *Proc. 9th International Symposium on Robot Control*, Gifu, Japan, September 2009.

[39] M. Schuresko and J. Cortes, "Distributed motion constraints for algebraic connectivity of robotic networks," *Journal of Intelligent and Robotic Systems*, vol. 56, no. 1-2, pp. 99–126, September 2009.

[40] ——, "Distributed tree rearrangements for reachability and robust connectivity," in *Hybrid Systems: Computetation and Control*, ser. Lecture Notes in Computer Science. Springer, 2009, vol. 5469, pp. 470–474.

[41] D. P. Spanos and R. M. Murray, "Motion planning with wireless network constraints," in *Proc. 2005 American Control Conference*, Portland, OR, June 2005, pp. 87–92.

[42] E. Stump, A. Jadbabaie, and V. Kumar, "Connectivity management in mobile robot teams," in *Proc. IEEE International Conference on Robotics and Automation*, Pasadena, CA, May 2008, pp. 1525–1530.

[43] M. M. Zavlanos, M. B. Egerstedt, and G. J. Pappas, "Graph theoretic connectivity control of mobile robot networks," *Proceedings of the IEEE: Special Issue on Swarming in Natural and Engineered Systems*, vol. 99, no. 9, pp. 1525–154, September 2011.

[44] A. Ganguli, J. Cortes, and F. Bullo, "Multirobot rendezvous with visibility sensors in nonconvex environments," *IEEE Transactions on Robotics*, vol. 25, no. 2, pp. 340–352, 2009.

[45] J. Cortes, S. Martinez, and F. Bullo, "Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions," *IEEE Transactions on Automatic Control*, vol. 51, no. 8, pp. 1289–1298, 2006.

[46] M. M. Zavlanos, H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Hybrid control for connectivity preserving flocking," *IEEE Transactions on Automatic Control*, vol. 54, no. 12, pp. 2869–2875, December 2009.

[47] D. V. Dimarogonas and K. H. Johansson, "Decentralized connectivity maintenance in mobile networks with bounded inputs," in *Proc. IEEE International Conference on Robotics and Automation*, Pasadena, CA, May 2008, pp. 1507–1512.

[48] A. Ajorlou and A. G. Aghdam, "A class of bounded distributed controllers for connectivity preservation of unicycles," in *Proc. 49th IEEE Conference on Decision and Control*, Atlanta, GA, December 2010, pp. 3072–3077.

[49] A. Ajorlou, A. Momeni, and A. G. Aghdam, "A class of bounded distributed control strategies for connectivity preservation in multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 12, pp. 2828–2833, December 2010.

[50] M. A. Hsieh, A. Cowley, V. Kumar, and C. Taylor, "Maintaining network connectivity and performance in robot teams," *Journal of Field Robotics*, vol. 25, no. 1-2, pp. 111–131, 2008.

[51] A. R. Wagner and R. C. Arkin, "Communication-sensitive multi-robot reconnaissance," in *Proc. IEEE International Conference on Robotics and Automation*, New Orleans, LA, 2004, pp. 2480–2487.

[52] M. Powers and T. Balch, "Value-based communication preservation for mobile robots," in *Proc. 7th International Symposium on Distributed Autonomous Robotic Systems*, Toulouse, France, 2004.

[53] Y. Mostofi, "Decentralized communication-aware motion planning in mobile networks: An information-gain approach," *Journal of Intelligent and Robotic Systems*, vol. 56, no. 1–2, pp. 233–256, 2009.

[54] P. Flocchini, G. Prencipe, N. Santoro, and P. Widmayer, "Gathering of asynchronous oblivious robots with limited visibility," *Theoretical Computer Science*, vol. 337, no. 1-3, pp. 147–168, 2005.

[55] R. C. Arkin and J. Diaz, "Line-of-sight constrained exploration for reactive multiagent robotic teams," in *Proc. 7th International Workshop on Advanced Motion Control*, Maribor, Slovenia, 2002, pp. 455–461.

[56] S. O. Anderson, R. Simmons, and D. Goldberg, "Maintaining line-of-sight communications networks between planetray rovers," in *Proc. 2003 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Las Vegas, NV, 2003, pp. 2266–2272.

[57] G. Hollinger and S. Singh, "Multi-robot coordination with periodic connectivity," in *Proc. IEEE International Conference on Robotics and Automation*, 2010, (accepted).

[58] M. M. Zavlanos, "Synchronous rendezvous of very-low-range wireless agents," in *Proc. of the 49th IEEE Conference on Decision and Control*, Atlanta, GA, December 2010, pp. 4740–4745.

[59] T. Gustavi, D. V. Dimarogonas, M. Egerstedt, and X. Hu, "Sufficient conditions for connectivity maintenance and rendezvous in leader-follower networks," *Automatica*, vol. 46, no. 1, pp. 133–139, January 2010.

[60] D. Tardioli, A. R. Mosteo, L. Riazuelo, J. L. Villarroel, and L. Montano, “Enforcing network connectivity in robot team missions,” *International Journal of Robotics Research*, (to appear).

[61] N. Michael, M. M. Zavlanos, V. Kumar, and G. J. Pappas, “Maintaining connectivity in mobile robot networks,” in *Experimental Robotics*, ser. Tracts in Advanced Robotics. Springer, 2009, pp. 117–126.

[62] H. Lundgren, E. Nordstrom, and C. Tschudin, “The gray zone problem in ieee 802.11b based ad hoc networks,” *ACM SIGMOBILE Mobile Computing and Communications Review*, vol. 6, no. 3, pp. 104–105, July 2002.

[63] M. M. Zavlanos, A. Ribeiro, and G. J. Pappas, “Network integrity in mobile robotic networks,” *IEEE Transactions on Automatic Control*, vol. 58, no. 1, pp. 3–18, January 2013.

[64] D. DeCouto, D. Aguayo, J. Bicket, and R. Morris, “A high-throughput path metric for multihop wireless routing,” in *Proc. ofInternational ACM Conference on Mobile Computing and Networking*, San Diego, CA, September 2006, pp. 134–146.

[65] A. Ribeiro, Z.-Q. Luo, N. D. Sidiropoulos, and G. B. Giannakis, “Modelling and optimization of stochastic routing for wireless multihop networks,” in *Proc. 26th Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM)*, Anchorage, Alaska, May 2007, pp. 1748–1756.

[66] A. Ribeiro, N. D. Sidiropoulos, and G. B. Giannakis, “Optimal distributed stochastic routing algorithms for wireless multihop networks,” *IEEE Transactions on Wireless Communications*, vol. 7, no. 11, pp. 4261–4272, November 2008.

[67] M. J. Neely, “Universal scheduling for networks with arbitrary traffic, channels, and mobility,” in *Proc. 49th IEEE Conference on Decision and Control*, Altanta, GA, December 2010, pp. 1822–1829.

[68] A. Ghaffarkhah and Y. Mostofi, “Communication-aware motion planning in mobile networks,” *IEEE Transactions on Automatic Control, special issue on Wireless Sensor and Actuator Networks*, vol. 56, no. 10, pp. 2478–248, October 2011.

[69] Y. Yan and Y. Mostofi, “Robotic router formation in realistic communication environments,” *IEEE Transactions on Robotics*, vol. 28, no. 4, pp. 810–827, August 2012.