Three-Dimensional Multirobot Formation Control for Target Enclosing

Miguel Aranda, Gonzalo López-Nicolás, Carlos Sagüés and Michael M. Zavlanos

Abstract— **This paper presents a novel method that enables a team of aerial robots to enclose a target in 3D space by attaining a desired geometric formation around it. We propose an approach in which each robot obtains its motion commands using measurements of the relative position of the other agents and of the target, without the need for a central coordinator. As contribution, our method permits any desired 3D target enclosing configuration to be defined, in contrast with the planar circular patterns commonly encountered in the literature. The proposed control strategy relies on the minimization of a cost function that captures the collective motion objective. In our method, the robots do not need to use a common reference frame. This coordinate independence is achieved through the introduction in the cost function of a rotation matrix computed locally by each robot. We prove that our motion controller is exponentially stable, and illustrate its performance through simulations.**

I. Introduction

The use of multiple mobile robots to collectively carry out tasks of interest is a prominent topic within the robotics research community. In particular, the problem of enclosing a target using a multirobot team has received significant attention due to the relevance and diversity of its applications, which include entrapment of an unfriendly element, escorting missions, or collective perception of a given location in an environment.

The target enclosing problem pertains to the generic field of multiagent formation control [1], a wide area of research which encompasses very diverse scenarios. A common element of the existing works in this field is that a group of robots are required to attain and/or maintain a geometric pattern typically expressed in terms of absolute agents positions [2], [3], relative interagent distances [4], [5] or relative interagent positions [6]–[8]. Circular formations, which are commonly employed for enclosing tasks, have received notable attention [9], [10]. The leader-follower formation control paradigm [11], [12], in which one particular robot acts as the reference for the others, is closely related to the problem considered in this paper. A growing number of works specifically address 3D formation control [13], [14], in which the higher dimensionality and the dynamics of aerial vehicles can pose additional challenges.

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Next, we discuss related works in which it is specifically required for a multirobot team to enclose a target. A null space-based controller was proposed in [15], while the control strategy in [16] is defined in cluster space. These two approaches are centralized and tackle escorting/entrapment missions. In [17], a distributed and reconfigurable method employing only local sensing (i.e. not requiring a global coordinate frame) and the velocity of the target is presented. The work [18] addresses target enclosing while considering changing topologies in the robots' interaction graph, presenting a distributed and coordinate-free approach, two properties also shared by the method in [19], which assumes the measurements used by the robots are anonymous. In all the above works, the robots are deployed in a circular formation around the target, whereas in [20] the use of elliptical orbits is proposed, in order to account for uncertainty in the target position estimations. The 3D instance of the enclosing problem has also been addressed: in [21], target encirclement in 3D space is achieved via model predictive control, while in [22] a dynamic network topology is assumed. Recently, the work [23] extended [19] to deal with a 3D workspace. In these works [21]–[23], the robots achieve a planar circular configuration around the target.

The main contribution of our work lies in the fact that the enclosing of a target in 3D space is achieved with the robots attaining any desired spatial configuration. We argue that for the 3D enclosing task, a three dimensional enclosing pattern provides advantages when compared to a planar one. In particular, it reduces the size of the escape areas, defined as the spacing between the enclosing robots. The quality of the collective perception of the target obtained by the multirobot team is improved as well. In addition, it can be interesting to use spatial configurations where the distance from the target is not the same for all the robots. For instance, this flexibility enables to increase the number of enclosing robots while respecting given safety distances between them.

Our method relies on a formation control formulation based on relative position measurements. Each robot computes its motion command using the measured positions of the other robots and of the target, with the goal of minimizing a cost function that encapsulates the target enclosing objective. The method brings the robotic team to a formation having an arbitrary rotation. This is consistent with the objective of an enclosing task, for which the rotation of the multirobot pattern surrounding the target is irrelevant. In addition, and contrary to the existing relative position-based formation control approaches, our method does not require a common reference frame for the robots. This is a relevant fact since it means that no additional sensors are needed.

Besides, the robots do not rely on the availability of a global reference which could be considered superfluous taking into account the essence of an enclosing mission, in which the focus is on the target.

In the approach we propose, the agents use global information. In this respect, we observe that for the enclosing of a single target, the number of robots required is typically modest, as illustrated by the works cited above. Therefore, the scalability of the system is not as important for this task as it is for those problems where large multirobot teams may be involved. We formally prove that the proposed control method is exponentially stable, and analyze its characteristics regarding collision avoidance and connectivity maintenance.

II. Problem formulation

Consider a group of $N-1$ robots in $\mathbb{R}^3$. Each robot is identified by an index $i \in 1,...,N-1$ and its dynamics can be expressed through a single integrator model, i.e. it satisfies:

$$\dot{\mathbf{q}}_{\mathbf{i}} = \mathbf{u}_{\mathbf{i}}, \tag{1}$$

where $\mathbf{q_i} \in \mathbb{R}^3$ denotes the position vector of robot i and $\mathbf{u_i} \in \mathbb{R}^3$ is its control input. Let us denote as $\mathbf{q_N} \in \mathbb{R}^3$ the position of the target, i.e. the entity which the multirobot team is tasked to enclose. The robots and target positions are expressed in an arbitrary global reference frame.

We define a desired configuration, or formation shape, by a certain, fixed, reference layout of the $N-1$ robots in their configuration space. We consider the desired configuration as encoded through a set of inter-robot relative position vectors. Thus, let us denote as $\mathbf{c_{ij}}, \in \mathbb{R}^3 \;\forall i,j \in 1,...,N-1$, the vector from i to j in the reference layout that defines the desired configuration. We then consider that the robots are in the desired configuration if this layout has been achieved, up to a rotation and translation common to all the robots. Let us define the desired vectors from the target to each of the $N-1$ robots as $\mathbf{c_{Ni}} \;\forall i \in 1,...,N-1$, and consider that the desired position of the target is in the centroid of the desired configuration, i.e. $\sum_{i \in 1,...,N-1} \mathbf{c_{Ni}} = \mathbf{0}$. This paper addresses the following problem:

Problem 1: Given an initial configuration in which the robots and the target are in arbitrary positions, the robots are required to reach a set of final positions such that the robotic group is in the desired configuration and the target is in the centroid of the group.

Each robot is assumed to be able to compute, either through sensing or communications with the other robots, an estimation of the relative position of every other robot in the group and of the target. This is the only information used by the proposed control strategy, which is described in the following section.

III. Target Enclosing strategy

This section describes the proposed multirobot control strategy to carry out the target enclosing task. Let us define the following cost function:

$$\gamma = \sum_i \sum_j ||\mathbf{q_{ij}} - \mathbf{R}\mathbf{c_{ij}}||_F^2, \tag{2}$$

where the the sums are carried out over the set $1,...,N$, and the subscript F denotes the Frobenius norm, $\mathbf{q_{ij}} = \mathbf{q_i} - \mathbf{q_j}$, and $\mathbf{R} \in SO(3)$ is a rotation matrix. The cost function is a sum of squared distances that expresses how separated the set of robots is from achieving the desired configuration surrounding the target. The introduction in (2) of the rotation matrix $\mathbf{R}$ makes our control method independent of a global coordinate system, as discussed below. We compute this matrix so as to minimize γ. For this, let us define the following matrices obtained by stacking the interagent position vectors:

$$\begin{aligned} \mathbf{Q} &= [\mathbf{q_{11}}...\mathbf{q_{1N}}\; \mathbf{q_{21}}...\mathbf{q_{2N}}...\mathbf{q_{N1}}...\mathbf{q_{NN}}]^T \\ \mathbf{C} &= [\mathbf{c_{11}}...\mathbf{c_{1N}}\; \mathbf{c_{21}}...\mathbf{c_{2N}}...\mathbf{c_{N1}}...\mathbf{c_{NN}}]^T. \end{aligned} \tag{3}$$

The size of $\mathbf{Q}$ and $\mathbf{C}$ is $N^2 \times 3$. It turns out that we can find the rotation matrix that minimizes γ using the Kabsch algorithm [24], which employs Singular Value Decomposition (SVD) to compute the rotation that aligns two sets of vectors with minimum quadratic error. In particular, let us define the matrix $\mathbf{A} = \mathbf{C}^T\mathbf{Q}$, of size 3×3. An analytical expression for the solution to this problem can be given as follows:

$$\mathbf{R} = (\mathbf{A}^T\mathbf{A})^{1/2}\mathbf{A}^{-1}, \tag{4}$$

which is not always applicable (e.g. if $\mathbf{A}$ is singular). In contrast, the method in [24] always gives a solution. Specifically, denoting the SVD of $\mathbf{A}$ as follows: $\mathbf{A} = \mathbf{USV}^T$, the rotation matrix we look for is given by:

$$\mathbf{R} = \mathbf{VDU}^T = \mathbf{V}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & d \end{pmatrix}\mathbf{U}^T, \tag{5}$$

where $d = sign(det(\mathbf{VU}^T))$. This SVD-based method is known to give a unique solution unless $rank(\mathbf{A}) < 2$ or the smallest singular value of $\mathbf{A}$ is degenerate [25]. Let us note that, for efficiency, $\mathbf{A}$ can be computed as $\mathbf{A} = \mathbf{C_o}^T\mathbf{Q_o}$, where $\mathbf{Q_o} = [\mathbf{q_{1o}} \ldots \mathbf{q_{No}}]^T$, $\mathbf{C_o} = [\mathbf{c_{1o}} \ldots \mathbf{c_{No}}]^T$. $\mathbf{C_o}$ and $\mathbf{Q_o}$ are $N \times 3$ matrices, and $\mathbf{q_o}$ and $\mathbf{c_o}$ are defined as the centroids of the current and desired sets of robot positions, respectively.

We define the control law for each agent i as follows:

$$\dot{\mathbf{q}}_{\mathbf{i}} = K_c(\mathbf{q_{Ni}} - \mathbf{R}\mathbf{c_{Ni}}), \tag{6}$$

where K_c is a positive control gain. It is important to note that each robot can compute its control input in its own local coordinate frame. Observe first that $\mathbf{q_{ij}}$ are relative measurements, and therefore, there is no need for a common coordinate origin for the robots. The same holds for the measurement of the centroid of the robots' positions. Furthermore, the specific orientation of each robot's reference frame is irrelevant. In order to illustrate that a common reference is not needed, let us denote as $\mathbf{P_i} \in SO(3)$ the relative rotation matrix between the global frame and the local frame in which i operates, i.e. $\mathbf{q_i^P} = \mathbf{P_i}\mathbf{q_i}$, with $\mathbf{q_i^P}$ being the position of i in a frame centered in the global origin and aligned with the local frame. It can be readily observed that the rotation matrix computed by robot i, which minimizes (2), is equal to $\mathbf{P_i R}$. Thus, the control law computed in the local frame has the form (6) when expressed in the global one.

Note, as well, that in a scenario where the robots sense locally only partial information of the system and obtain the rest through communications, a global reference is not needed either. Indeed, assume that robots i and j can sense and communicate with each other. Clearly, i can compute the rotation between its local frame and $j's$ local frame, since i knows the vector $\mathbf{q_{ij}}$ expressed in both frames. Thus, i can compute the positions, in its own frame, of the robots sensed by j. By extension, considering connected sensing and communication topologies, it can do so for all the robots.

IV. Stability analysis

This section analyzes the stability of the target enclosing strategy. Let us make the following assumption:

A1: $rank(\mathbf{A}) > 1$ and the smallest singular value of $\mathbf{A}$ is nondegenerate at every time instant.

We obtain the following result:

Proposition 1: If **A1** holds, given a static target and a set of robots which evolve according to the control law (6), the multirobot team converges exponentially to the desired configuration, with the target at the centroid of the set.

Proof: The proof proceeds by showing that $\dot{\mathbf{R}} = \mathbf{0}\ \forall t$ and inferring exponential stability from that fact. Observe first that in our controller (6) it is satisfied that $\mathbf{R}$, obtained using (5), is such that it minimizes γ. Therefore, the gradient of γ with respect to the rotation matrix $\mathbf{R}$ must be null. Let us express (2) as $\gamma = \sum_i \sum_j \gamma_{ij}$, with $\gamma_{ij} = ||\mathbf{q_{ij}} - \mathbf{R}\mathbf{c_{ij}}||_F^2$, and note that $\mathbf{A} = \sum_i \sum_j \mathbf{A_{ij}}$, where $\mathbf{A_{ij}} = \mathbf{c_{ij}}\mathbf{q_{ij}}^T$. From [26], where the gradients are derived for a function analogous to γ_{ij}, we have:

$$\nabla_{\mathbf{R}} \gamma_{ij} = \mathbf{R}^T \mathbf{A_{ij}}^T - \mathbf{A_{ij}}\mathbf{R}. \quad (7)$$

We can directly compute $\nabla_{\mathbf{R}}\gamma$, and impose the condition that it has to be null. This gives:

$$\nabla_{\mathbf{R}} \gamma = \mathbf{R}^T \mathbf{A}^T - \mathbf{A}\mathbf{R} = \mathbf{0}. \quad (8)$$

Considering that $\mathbf{R}$ is differentiable, we obtain, by differentiating (8) with respect to time:

$$\dot{\mathbf{R}}^T \mathbf{A}^T + \mathbf{R}^T \dot{\mathbf{A}}^T - \dot{\mathbf{A}}\mathbf{R} - \mathbf{A}\dot{\mathbf{R}} = \mathbf{0}. \quad (9)$$

In order to find $\dot{\mathbf{R}}$ from (9), we will first compute $\dot{\mathbf{A}}$. For this, observe that we can directly obtain from (6):

$$\dot{\mathbf{q}}_{\mathbf{ij}}(t) = \dot{\mathbf{q}}_{\mathbf{i}}(t) - \dot{\mathbf{q}}_{\mathbf{j}}(t) = -K_c[\mathbf{q_{ij}}(t) - \mathbf{R}(t)\mathbf{c_{ij}}]. \quad (10)$$

Notice that this also holds for $i = N$ or $j = N$. Stacking these vectors, we get:

$$\dot{\mathbf{Q}} = -K_c(\mathbf{Q} - \mathbf{C}\mathbf{R}^T). \quad (11)$$

Thus:

$$\dot{\mathbf{A}} = \mathbf{C}^T \dot{\mathbf{Q}} = -K_c(\mathbf{A} - \mathbf{C}^T\mathbf{C}\mathbf{R}^T). \quad (12)$$

Substitution of (12) in (9) yields:

$$\dot{\mathbf{R}}^T \mathbf{A}^T - K_c \mathbf{R}^T \mathbf{A}^T + K_c \mathbf{R}^T \mathbf{R} \mathbf{C}^T \mathbf{C}$$
$$+ K_c \mathbf{A}\mathbf{R} - K_c \mathbf{C}^T \mathbf{C} \mathbf{R}^T \mathbf{R} - \mathbf{A}\dot{\mathbf{R}} = \mathbf{0}. \quad (13)$$

Using in (13) that $\mathbf{R}$ is orthogonal and $\mathbf{AR}$ is symmetric (8), we finally get:

$$\dot{\mathbf{R}}^T \mathbf{A}^T - \mathbf{A}\dot{\mathbf{R}} = \mathbf{0}. \quad (14)$$

Observe that $\dot{\mathbf{R}} = \mathbf{0}$ is a solution to this equation. Let us now address the existence of nontrivial solutions. For this, we express the differentiation of a rotation matrix as $\dot{\mathbf{R}} = \mathbf{BR}$, where $\mathbf{B}$ is a skew-symmetric matrix [27]. Substituting in (14) gives:

$$\mathbf{R}^T \mathbf{B}^T \mathbf{A}^T - \mathbf{A}\mathbf{B}\mathbf{R} = \mathbf{0}. \quad (15)$$

If we substitute (5) and the SVD of $\mathbf{A}$ in (15), we get:

$$\mathbf{U}\mathbf{D}\mathbf{V}^T\mathbf{B}^T\mathbf{V}\mathbf{S}\mathbf{U}^T - \mathbf{U}\mathbf{S}\mathbf{V}^T\mathbf{B}\mathbf{V}\mathbf{D}\mathbf{U}^T = \mathbf{0}, \quad (16)$$

and using that $\mathbf{B}^T = -\mathbf{B}$, we obtain the expression:

$$\mathbf{DMS} + \mathbf{SMD} = \mathbf{0}, \quad (17)$$

where we define $\mathbf{M} = \mathbf{V}^T\mathbf{B}\mathbf{V}$, which is also skew-symmetric. Let us denote the entries of $\mathbf{S}$ and $\mathbf{M}$ as s_{ij} and m_{ij}, respectively. Thus, s_{ii} are the singular values of $\mathbf{A}$, in descending order. Then, (17) can be expressed as:

$$m_{12}(s_{11} + s_{22}) = 0$$
$$m_{13}(s_{11}d + s_{33}) = 0$$
$$m_{23}(s_{22}d + s_{33}) = 0. \quad (18)$$

We discuss next the cases in which the only solution to these equations is the trivial one, i.e. $\mathbf{M} = \mathbf{0}$ ($m_{12} = m_{13} = m_{23} = 0$). It is straightforward to see that if $d = 1$, there are no other solutions as long as $rank(\mathbf{A}) > 1$ (i.e. if $s_{11} > s_{22} > 0$). If $d = -1$, the trivial solution is unique as long as $rank(\mathbf{A}) > 1$ and $s_{33} \neq s_{22}$. Thus, due to assumption **A1**, we have that $\mathbf{M} = \mathbf{B} = \mathbf{0}$, and therefore, $\dot{\mathbf{R}} = \mathbf{0}$.

Let us denote the initial value of the rotation as $\mathbf{R_0}$. Since $\dot{\mathbf{R}} = \mathbf{0}$ (i.e. $\mathbf{R}$ is constant) then, from (10), we can write the evolution of the relative position vectors as follows:

$$\dot{\mathbf{q}}_{\mathbf{ij}}(t) = -K_c[\mathbf{q_{ij}}(t) - \mathbf{R}(t)\mathbf{c_{ij}}] = -K_c[\mathbf{q_{ij}}(t) - \mathbf{R}_0\mathbf{c_{ij}}], \quad (19)$$

which holds for $i, j = 1, ..., N$, i.e. for the vectors between two robots or between one robot and the target. Thus, we can conclude that the system converges exponentially to the desired configuration and the target is in the centroid of the attained multirobot formation. ■

Remark 1: Observe that the cases where we can ensure $\dot{\mathbf{R}} = \mathbf{0}$ correspond to the cases in which the Kabsch algorithm used to compute (5) gives a unique solution (see Section III). Let us discuss the cases where there are multiple solutions. The situation $rank(\mathbf{A}) \leq 1$ corresponds to a geometric configuration where the robot positions are in a line in space, while the other case is associated with degenerate singular values ($s_{33} = s_{22}$). Note that, even when the solution to (5) is not unique, the algorithm always outputs a valid solution, i.e. a rotation matrix that globally minimizes γ [25]. As a consequence, we observe that for every pair i and j, $\mathbf{R}\mathbf{c_{ij}}$ is equal for all possible valid solutions of $\mathbf{R}$. Thus, our method performs equally well for any arbitrary current and desired configurations of the robots in 3D space. We illustrate this fact in simulation (Section VI).

V. Method properties

We provide in this section a discussion of a number of relevant issues concerning the proposed control method.

A. Collision avoidance

The proposed controller allows to predict collisions. To illustrate this, notice that the predicted evolution of the vector between robots i and j at a given initial instant t_0 has, from (19), the following form:

$$\mathbf{q_{ij}}(t) = \mathbf{q_{ij}}(t_0)e^{-K_c(t-t_0)} + \mathbf{R_0 c_{ij}}\left[1 - e^{-K_c(t-t_0)}\right]. \quad (20)$$

Thus, we directly see that the vector $\mathbf{q_{ij}}$ will become null at $t = t_0 + \ln(2)/K_c$ if it holds that $\mathbf{q_{ij}}(t_0)$ and $\mathbf{R_0 c_{ij}}$ are parallel and lie on opposite sides of the coordinate origin. This is explicitly captured in the two following conditions: $\mathbf{q_{ij}} \times \mathbf{R_0 c_{ij}} = \mathbf{0}$ and $\mathbf{q_{ij}}^T \mathbf{R_0 c_{ij}} < 0$. Every agent can evaluate these conditions for all other agents, and this predictive ability can facilitate the actual avoidance of the collisions. In practice, the geometry of the robots must be taken into account. If every robot is considered to be contained in a sphere of radius r centered in its coordinate origin, then a collision is predicted to occur if $||\mathbf{q_{ij}}(t)|| \leq 2r$ at some t. Let us outline one possible simple strategy to actually avoid the collisions, either between robots or between one robot and the target. This could be done by modifying the control gain K_c temporarily for the robots that predict a future collision. The resulting gain imbalance among the robots would then modify the rotation matrix computed by the group, which would in turn change the robots' trajectories and thus allow the collision to be avoided.

B. Connectivity maintenance

Consider a communications-based implementation of our approach, where the robots exchange their locally measured relative position information to gain global knowledge of the group. Then, the robots form a networked system, which we can model through a graph $\mathcal{G}_c = (\mathcal{V}, \mathcal{E}_c)$. The edges $\mathcal{E}_c$ express the presence or absence of a direct communication link between every pair of nodes (associated to robots) in $\mathcal{V}$. It is typical to use a proximity-based graph model [1], i.e. one where an edge exists between two robots if the distance that separates them is less than a given threshold. Assuming this model, let us define an initial graph $\mathcal{G}_c^0 = (\mathcal{V}, \mathcal{E}_c^0)$ and a connected, desired graph $\mathcal{G}_c^d = (\mathcal{V}, \mathcal{E}_c^d)$, considering the inter-robot distances in the initial and desired configurations, respectively. In particular, we define a communication radius R as the threshold for these proximity-based graphs. We assume the robots' sensing ranges are sufficient to ensure that the control task is fulfilled if $\mathcal{G}_c$ remains connected (e.g. considering a sensing graph, $\mathcal{G}_s$, equal to $\mathcal{G}_c$). Let us also disregard the effects in the control of the time delays associated with multi-hop communications in the network. This is a reasonable assumption considering that the number of robots in the system is typically small. Then, from equation (20), it is straightforward to find that the distance between every pair of robots at any instant satisfies:

$$||\mathbf{q_{ij}}(t)|| \leq \max(||\mathbf{q_{ij}}(t_0)||, ||\mathbf{c_{ij}}||), \quad (21)$$

where $||\mathbf{c_{ij}}||$ is the desired distance between the pair. Assume that $\mathcal{G}_c^0$ is a supergraph of $\mathcal{G}_c^d$, i.e. $\mathcal{E}_c^d \subseteq \mathcal{E}_c^0$. Thus, for every pair of robots such that $\{i, j\} \in \mathcal{E}_c^d$, it holds that $||\mathbf{q_{ij}}(t_0)|| < R$ and $||\mathbf{c_{ij}}|| < R$. Therefore, none of the edges in $\mathcal{G}_c^d$ are lost throughout the control execution, which guarantees that connectivity is maintained in the assumed scenario.

C. Formation stabilization

The proposed method can be used for standard formation stabilization tasks, by simply removing from the control law the dependencies on the target. Observe that (6) can be written as follows:

$$\dot{\mathbf{q}}_\mathbf{i} = K_c\left[\sum_j \mathbf{q_{ji}} + \mathbf{q_{Nj}} - \mathbf{R}(\sum_j \mathbf{c_{ji}} + \mathbf{c_{Nj}})\right]. \quad (22)$$

Keeping only the inter-robot vectors measured by i, we can define the following control law:

$$\dot{\mathbf{q}}_\mathbf{i} = K_c\left[\sum_j \mathbf{q_{ji}} - \mathbf{R}\sum_j \mathbf{c_{ji}}\right], \quad (23)$$

where now the sums are for $j = 1, ..., N-1$. It can be shown that this control law follows the negative gradient of γ with respect to $\mathbf{q_i}$. Thus, it brings the multirobot team to any arbitrarily specified configuration. Since (10) also holds when using (23), the convergence is exponential. An important advantage of this approach is that it does not require a global reference frame, which differentiates our method from existing formation stabilization works using relative position measurements.

VI. Simulations

This section presents results from simulations to illustrate the performance of the presented method. In the first example, a team of six robots is considered, and the desired configuration is an octahedron. The target in 3D space is static. Figure 1 illustrates how the robots converge exponentially to the desired configuration while enclosing the target. Observe that the 3D rotation of the enclosing multirobot configuration is arbitrary. The norms of the velocities of the robots, shown in Fig. 2, exhibit an exponential decay, as theoretically expected. We illustrate in Fig. 3 how the behavior of the inter-robot distances satisfies the equation (21), a fact that can be exploited so as to guarantee connectivity maintenance for the system (Section V-B). To illustrate that the performance of our method does not depend on the geometry (current or desired) of the robotic team's configuration, we show in Fig. 4 the results from an example in which the robots are initially on a plane and the desired configuration is a straight line with the target at its center.

Although we assumed thus far that the target was stationary, we observe that the proposed method can accommodate a scenario where the target moves. The multirobot system will be able to keep it enclosed as long as the maximum velocity achievable by the target is small compared to the maximum velocity achievable by the robots. The robots' ability to track the target's motion can be adjusted with

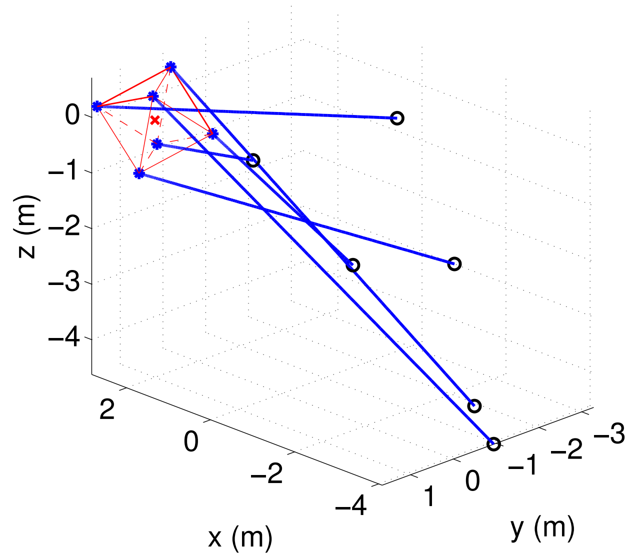

Fig. 1. Robot paths from arbitrary initial positions (circles) to the positions in an octahedron-shaped enclosing formation (stars) of a target (cross) situated at coordinates (2,1,0).

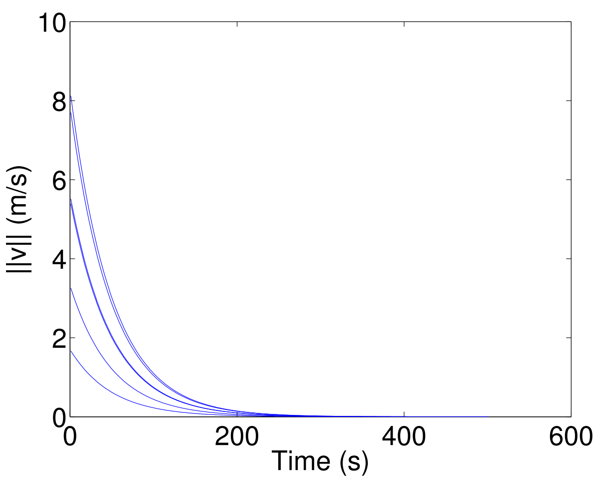

Fig. 2. Velocity norms for the octahedron-shaped configuration.

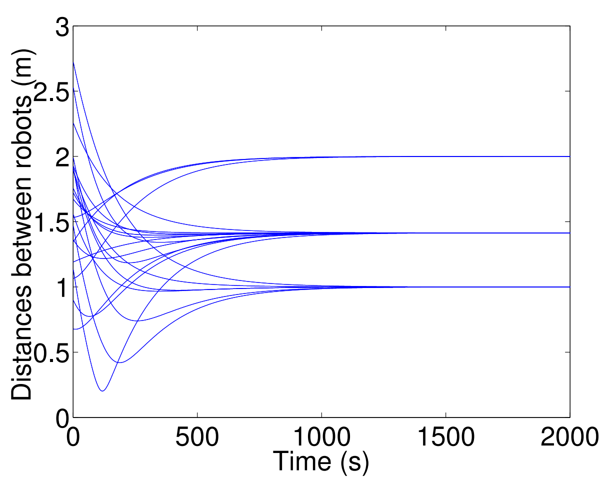

Fig. 3. Inter-robot distances for the octahedron-shaped configuration.

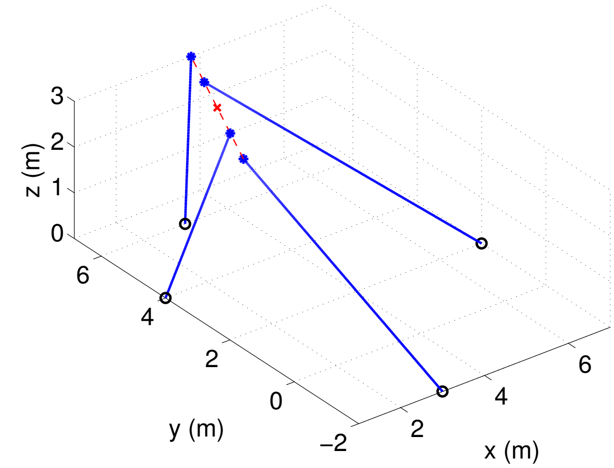

Fig. 4. Robot paths for four robots lying initially on a plane and forming a straight-line configuration centered on the target, showing the initial (circles) and final (stars) robot positions and the target position (cross).

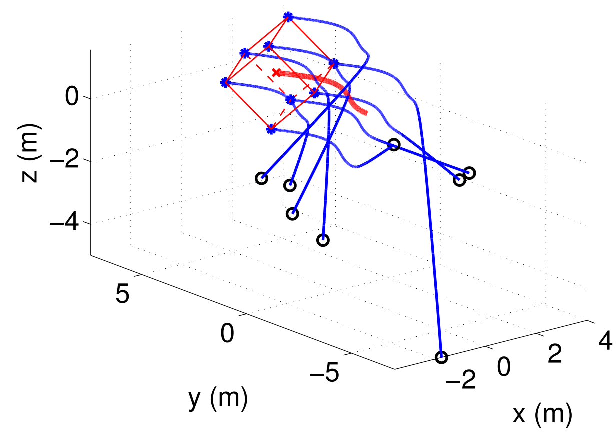

Fig. 5. Robot paths from arbitrary initial positions (circles) to a enclosing, cube-shaped formation (stars) of a target (cross) moving in a slowly varying sinusoidal fashion (thick line).

the gain K_c. We illustrate this behavior with a simulation example where a team of eight robots is considered and the desired configuration is a cube. The target in 3D space follows a slowly varying sinusoidal motion pattern. Figure 5 shows how the robots are able to track the motion of the target and keep it enclosed while approximately maintaining the desired configuration. Observe again that the 3D rotation of the multirobot configuration is arbitrary. The cost function γ, shown in Fig. 6, displays a non-vanishing error, since the specified geometric configuration is not attained exactly due to the persistent motion of the target. In addition, the robot velocities depicted in the same figure show how the robots track the target's motion. Notice that if the target is a friendly agent (as occurs in escorting missions), it may be able to communicate its velocity to the robots. In that case, it is straightforward in our method to subtract this velocity from the control law so that the target is always kept in the centroid of the multirobot formation.

Finally, we present, for illustration purposes, the results from an example where the robots maintain the enclosing of

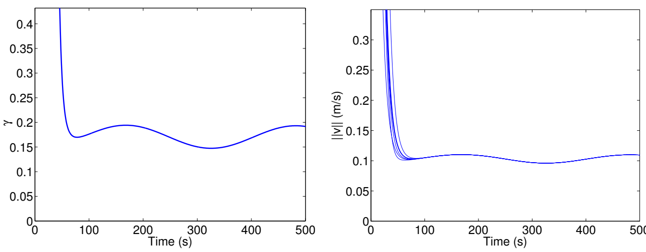

Fig. 6. Evolution of the cost function γ (left) and norms of the robots' velocities (right) for the cube-shaped enclosing pattern simulation example.

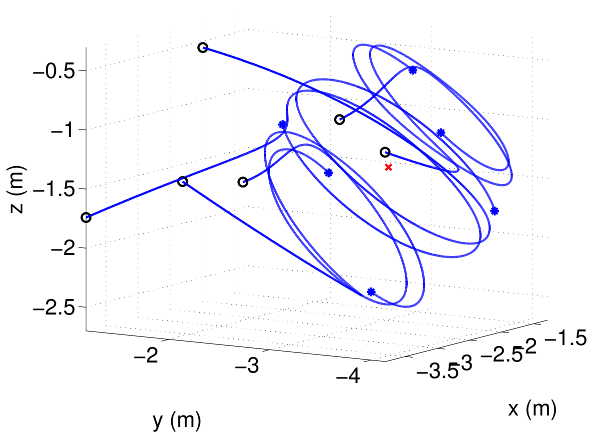

Fig. 7. Robot paths for an octahedron desired formation with an additive cyclic pursuit velocity component, showing the initial (circles) and final (stars) robot positions and the target position (cross).

the target while they gyrate around it. This is achieved in a simple manner by incorporating in (6) an additive velocity component proportional to the relative vector from robot i to $i+1$, modulo $N-1$ (i.e. a cyclic pursuit strategy). Figure 7 displays the robots' paths for an example using an octahedron formation. Further illustration of results from simulations can be found in the video that accompanies the paper.

VII. Conclusion

We have proposed a method to enclose a target in 3D space using a team of aerial robots. The approach has been shown to converge exponentially and to provide good performance and flexibility in the accomplishment of the required task. Even though single-target enclosing has been typically carried out employing modest numbers of robots, it would be interesting to make the use of information distributed in our approach, i.e. to make each robot interact only with a subset of the group. Results observed in simulation suggest that this would be feasible. One direction for our future work will be to formally address this problem. In addition, other interesting issues to consider are a deeper study of collision avoidance guarantees, a more realistic modeling of vehicle dynamics, and the effects in the control of time delays due to multi-hop communications.

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